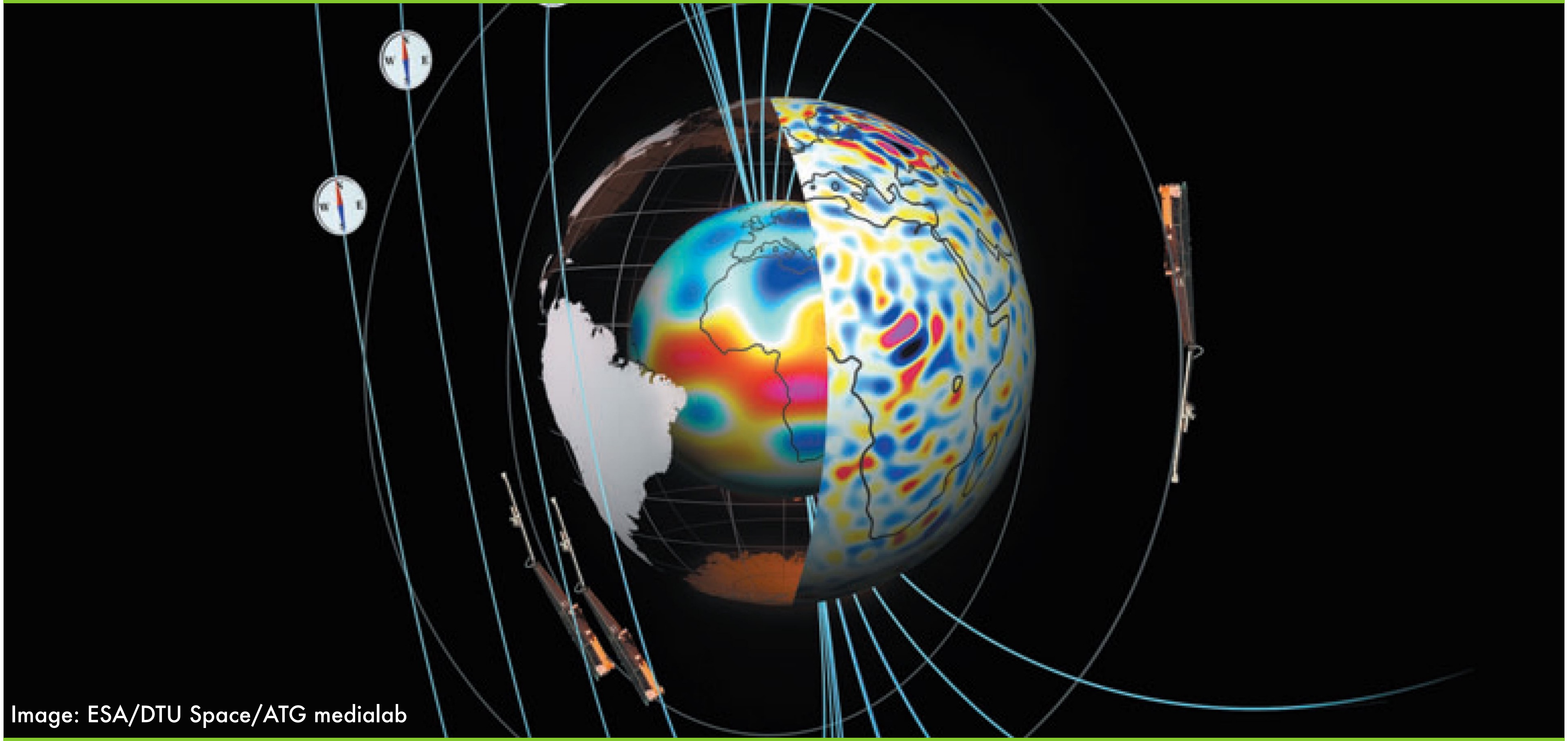
The Earth's Magnetic Field



Jon Mound

UNIVERSITY OF LEEDS

Lots of Questions

Lots of Questions

- How do you generate a (planetary) magnetic field?
- Has the Earth always had a magnetic field?
- Can the field flip? How do we know?
- Why does the field flip?
- What difference would it make if it did flip?
- Why might we think the field is about to flip?

Inside the Earth

- Thin rocky crust, generally 5-50 km thick
- Thick rocky mantle, 2900 km thick
- Liquid metal outer core 2260 km thick
- Solid metal inner core 1220 km radius

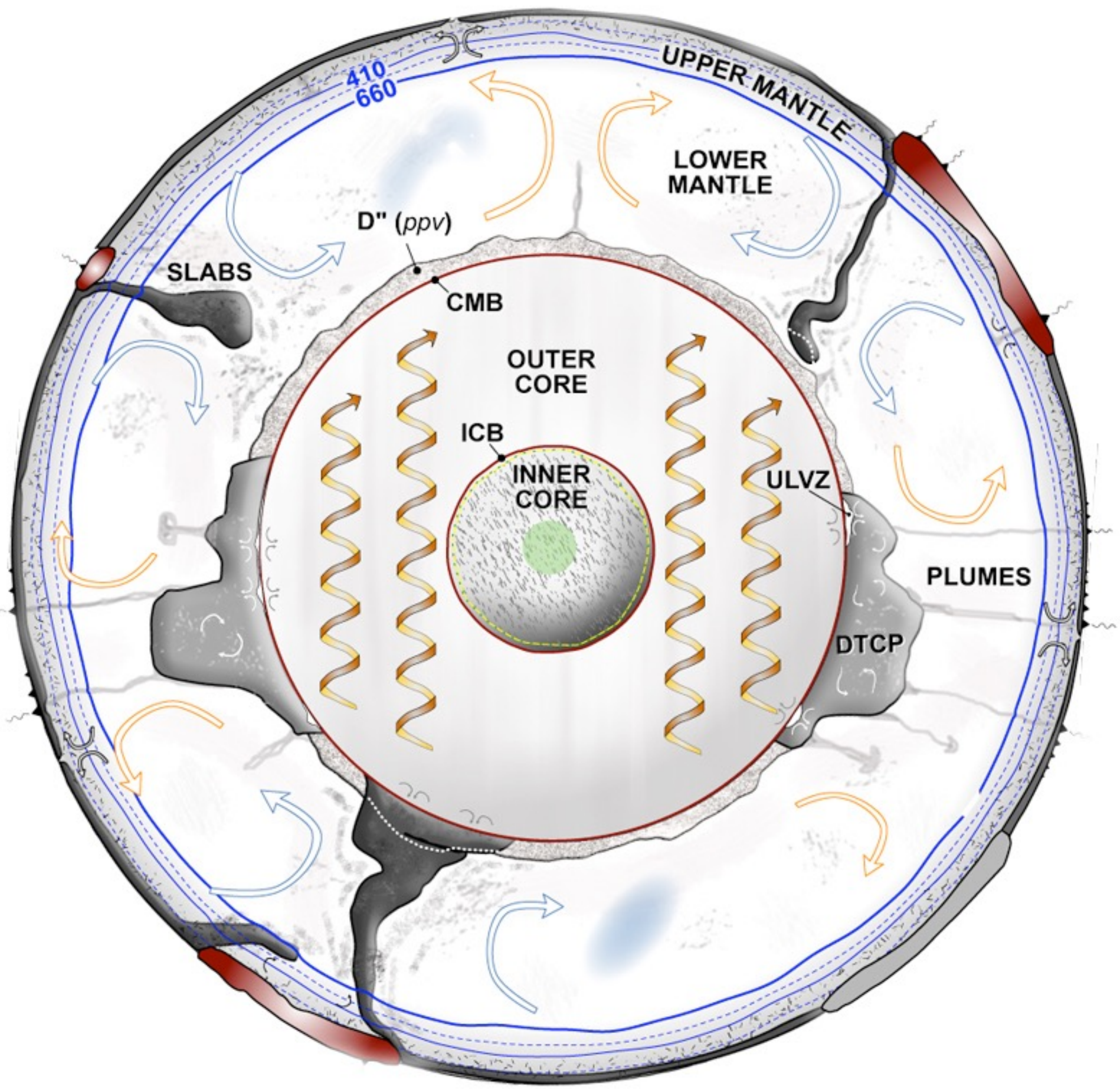
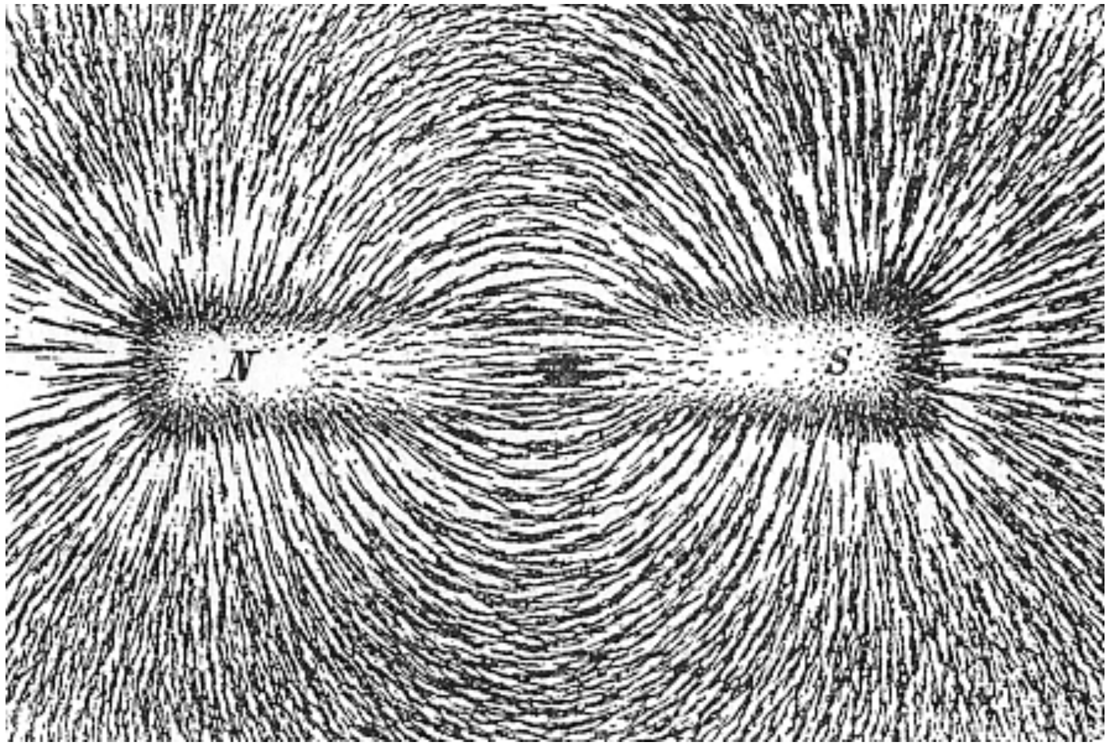


Image: Ed Garnero, Arizona State University

Where do Magnetic Fields Come From?

Where do Magnetic Fields Come From?

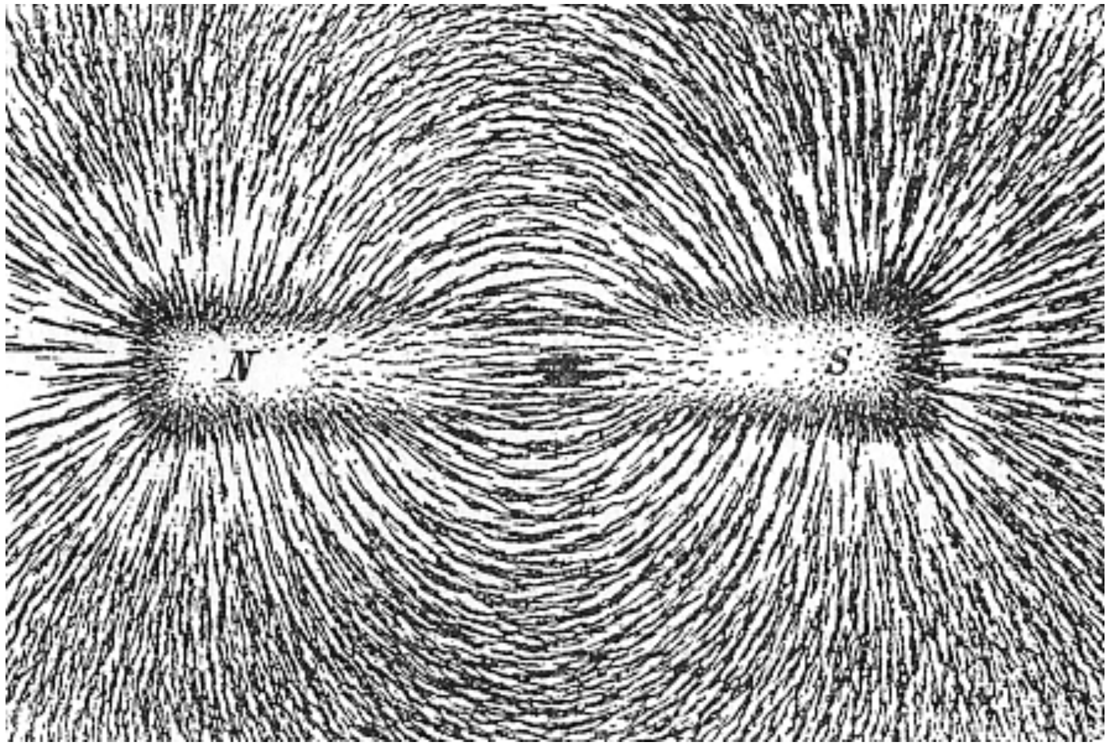
Image: wikipedia



- Magnets!

Where do Magnetic Fields Come From?

Image: wikipedia



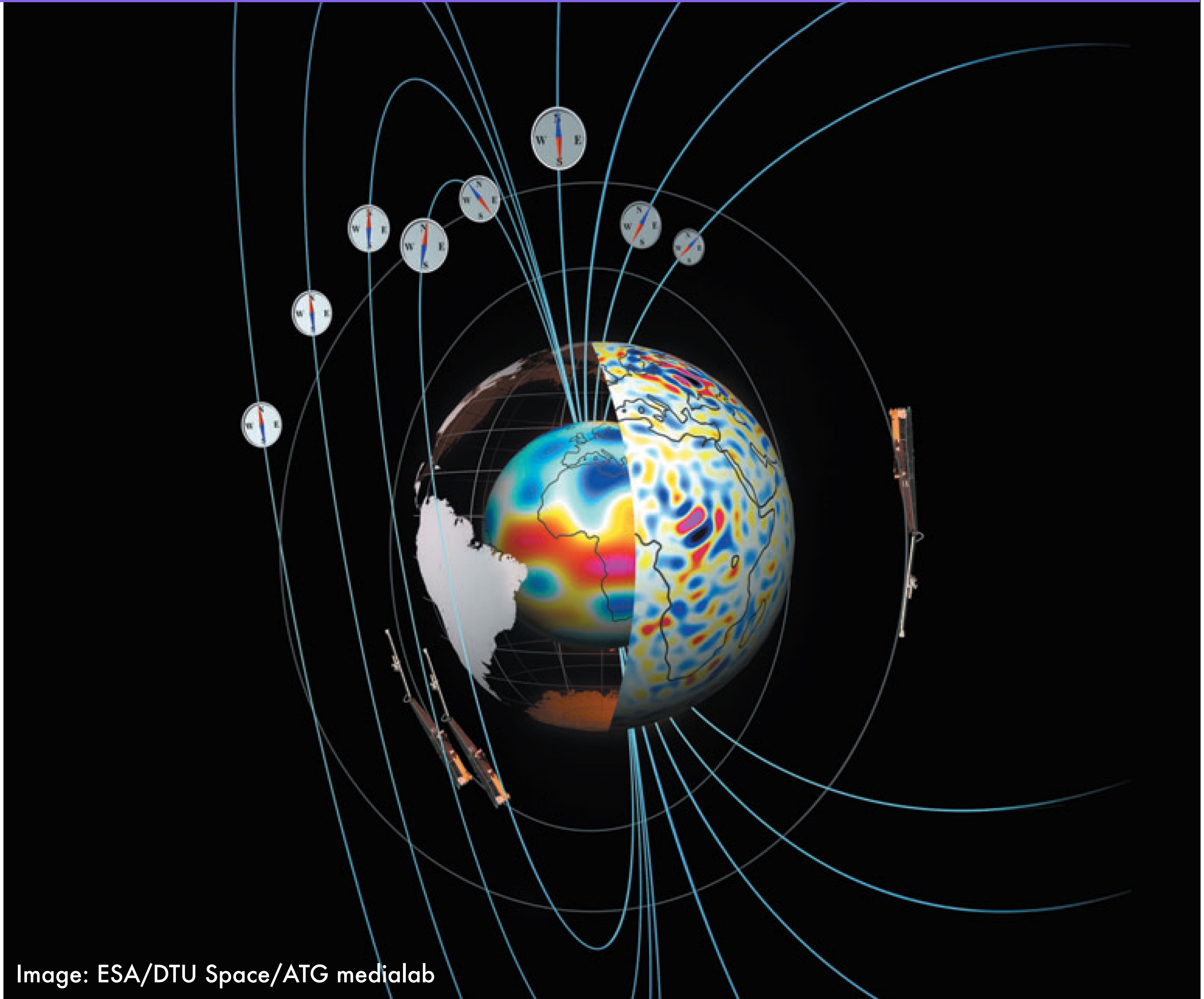
- Magnets!

- Electric Currents

Image: BBC

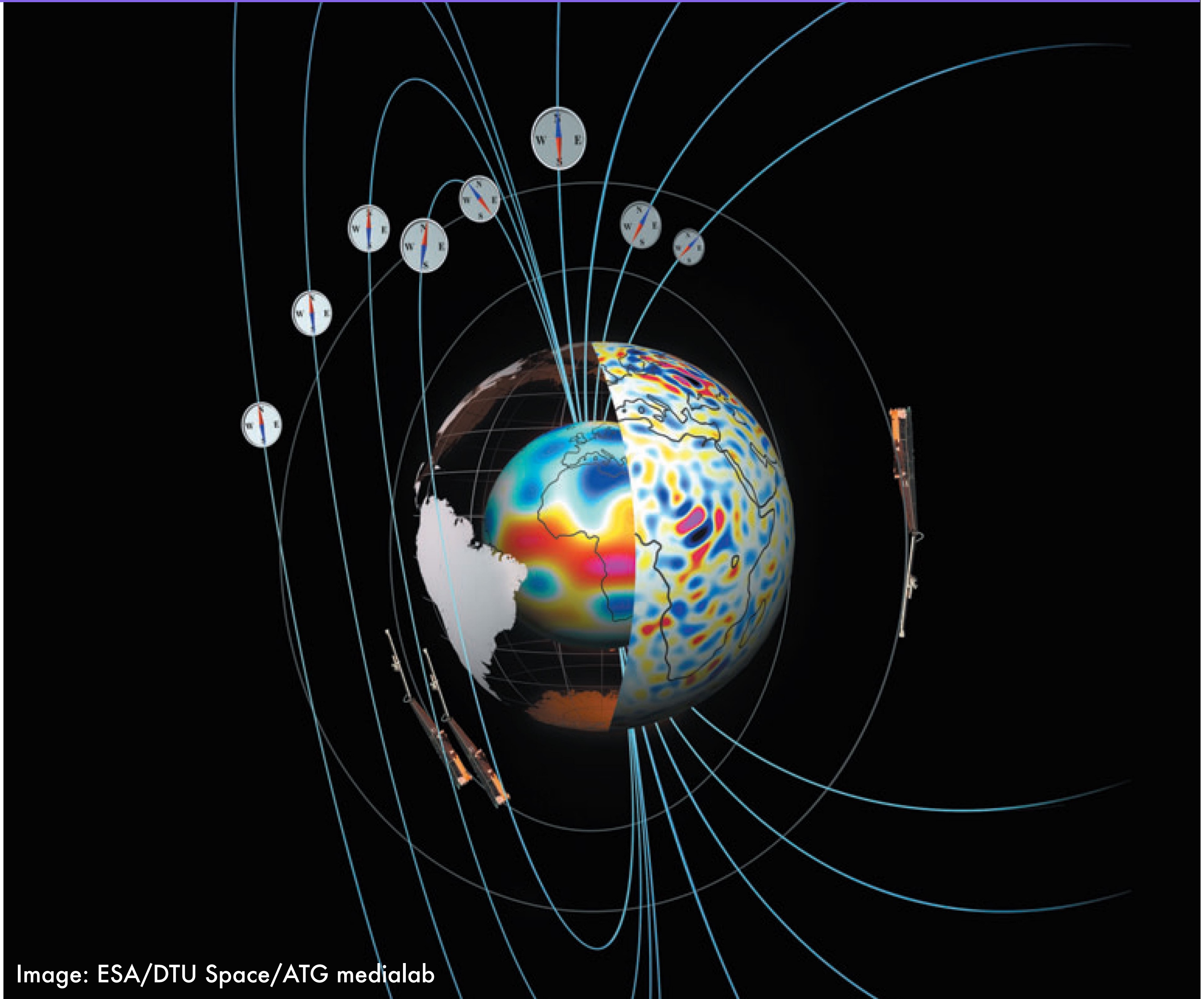
Permanent Magnets in the Earth

Certain rocks and minerals in the crust are magnetic.



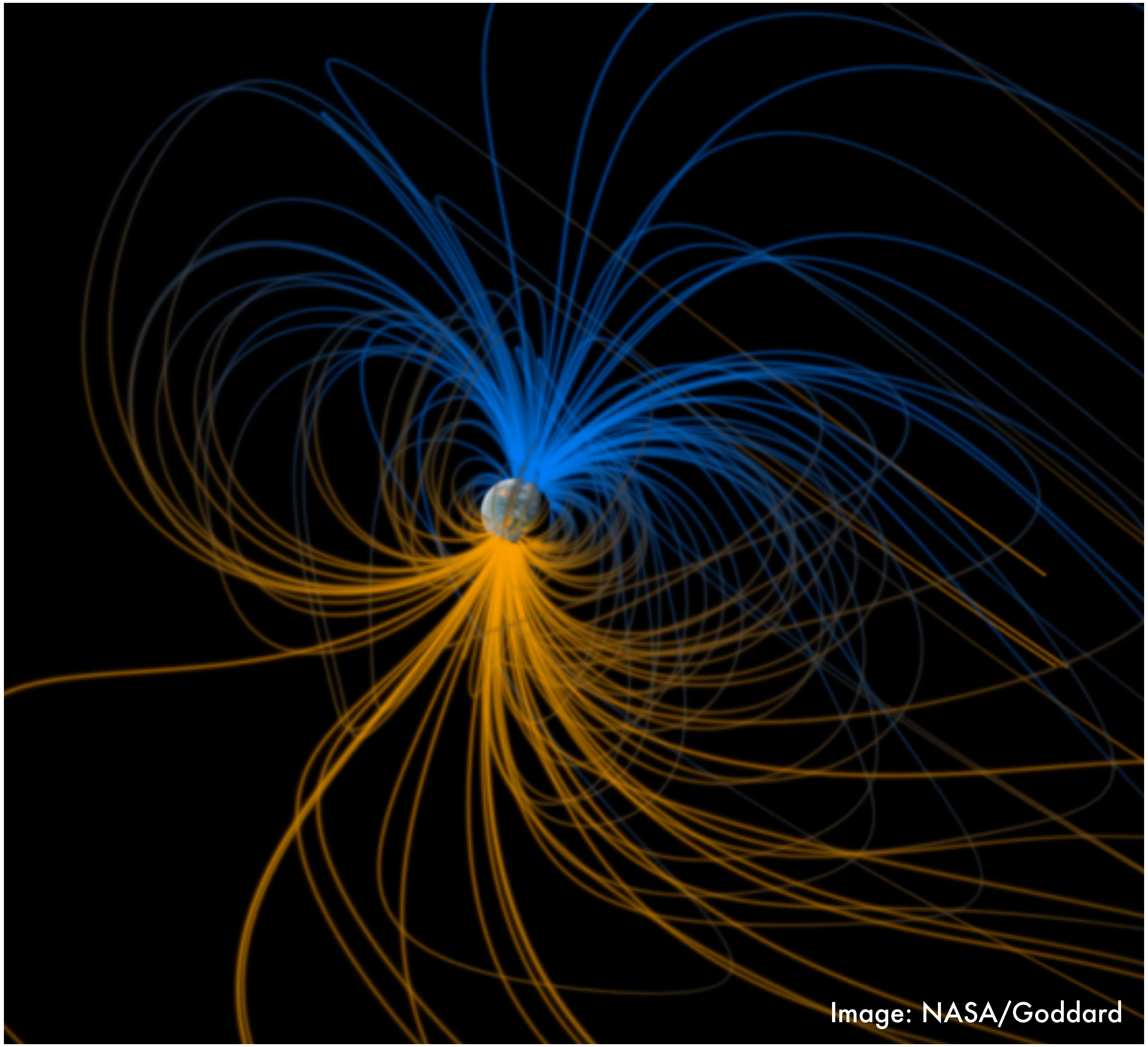
Electric Currents in the Earth

There are currents in the atmosphere, oceans, and core of the Earth.



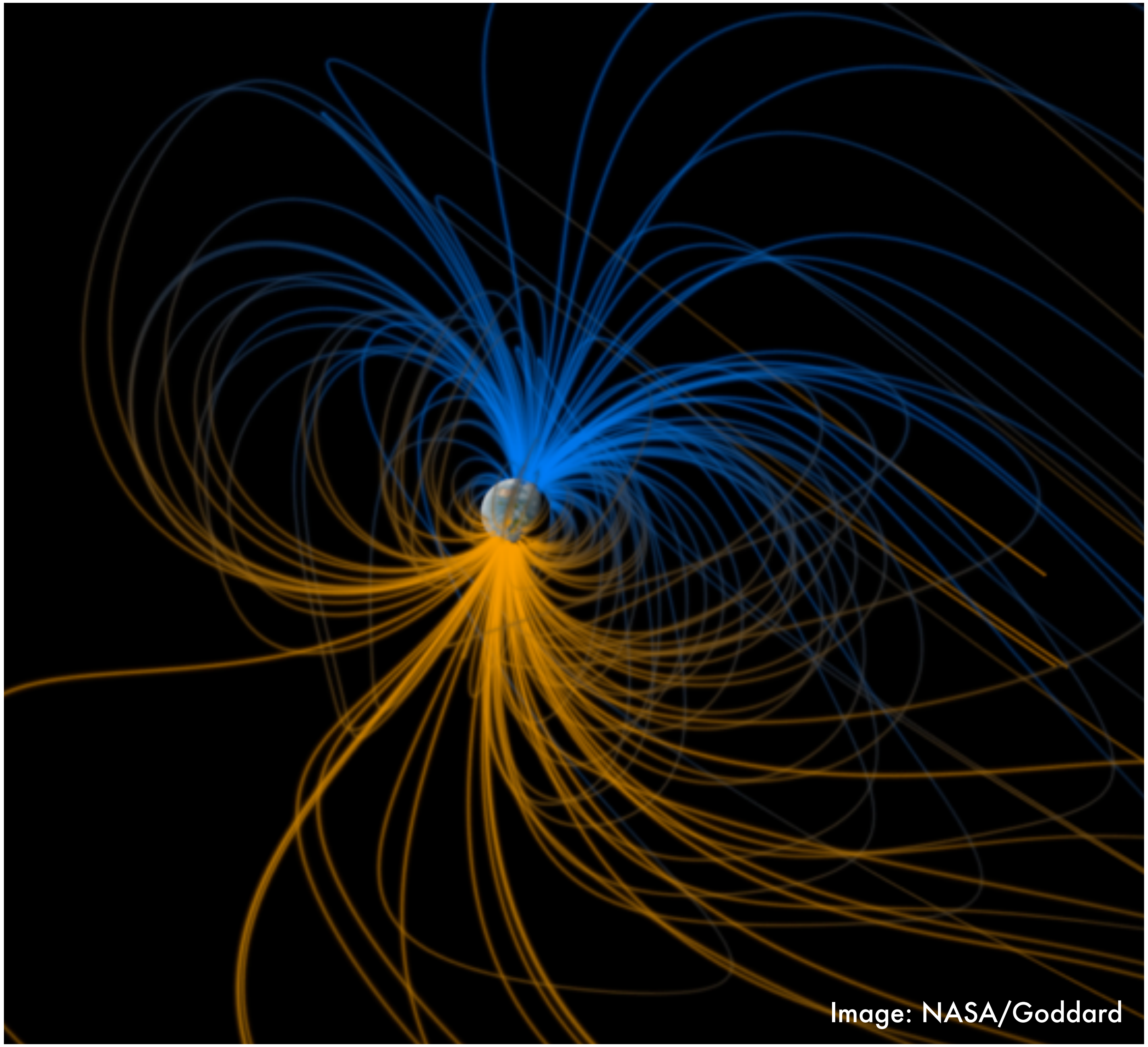
Earth's Global Field

- Field from the core
- Large-scale
- Long-lived
- Reverses!
- Reversing?



The Dynamo in Earth's Core

- What is a dynamo?
- Why do we "need" a geodynamo?
- How do we study the geodynamo?



What is a (geo)dynamo?

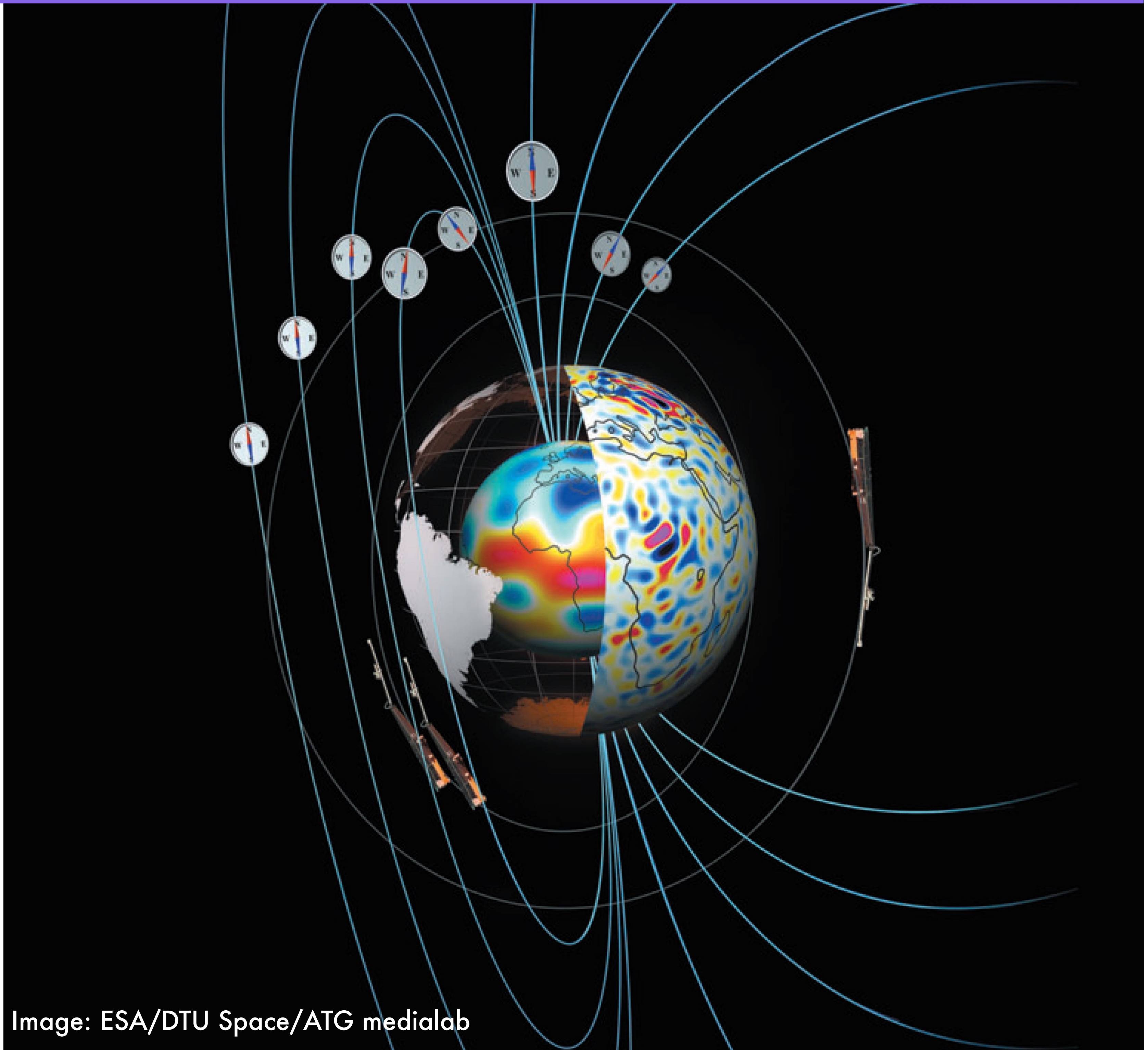
- A system that generates a magnetic field, converting kinetic energy to magnetic energy
- The geodynamo is self-exciting (continuous generation of magnetic field)
- The geodynamo arises from convection in the core

Why a Geodynamo?

- Earth's magnetic field is strong, large scale, long-lived, dynamic, reversing, and internal
- Earth's interior temperature is well above the Curie point
- Outer core is an electrically conductive, convecting, rotating fluid

How do we study the dynamo in Earth's core?

- Observations of current structure and dynamics.
- Satellite missions.
- Global network of ground-based observatories.

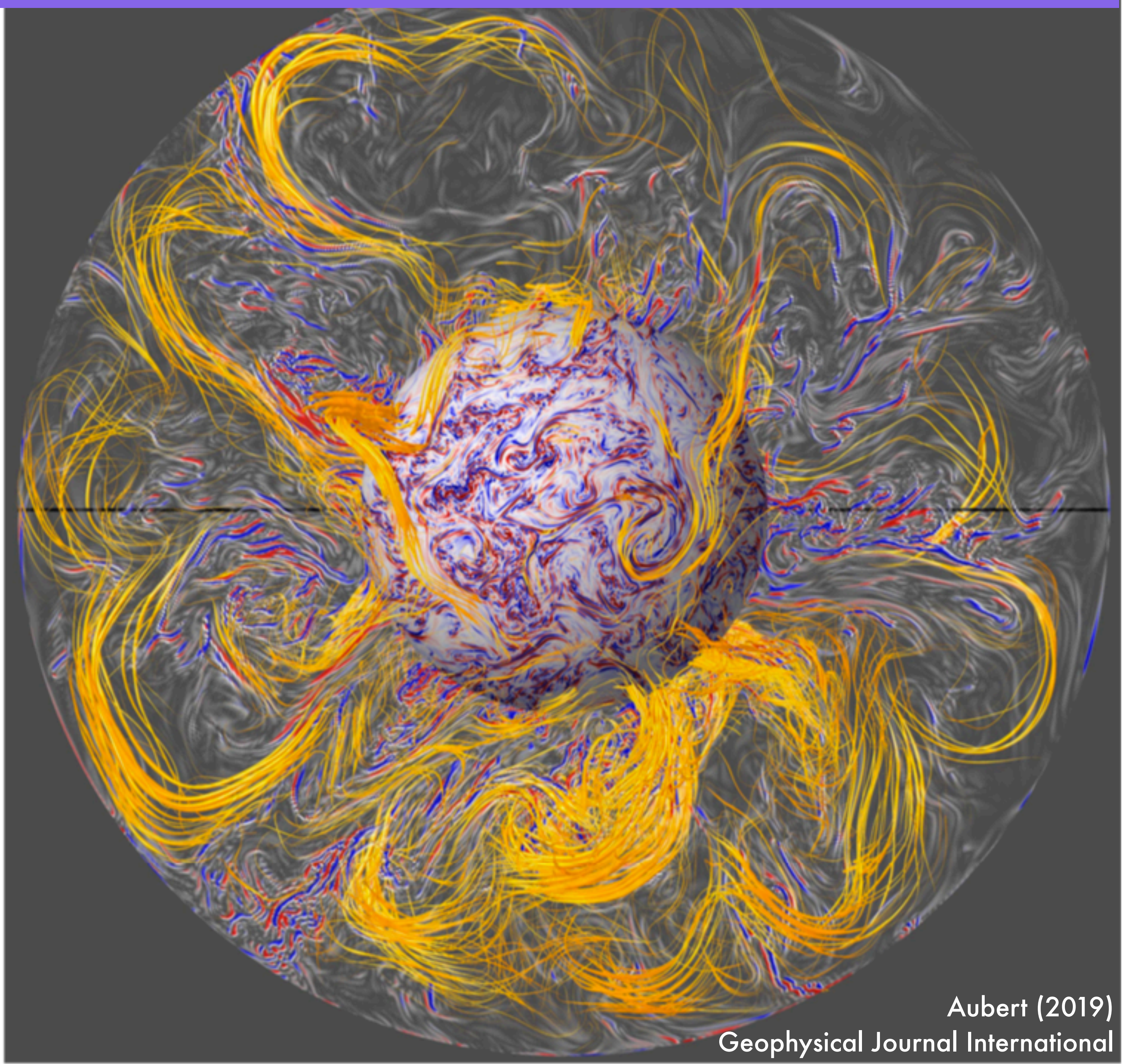


How do we study the dynamo in Earth's core?

- Certain rocks and minerals (and archaeological artefacts) are magnetic.
- Find them.
- Sample them.
- Measure them.
- Combine geological observations with magnetic measurements.

How do we study the dynamo in Earth's core?

- Computer simulations.
- Try out "lots of Earths."
- "See" time evolution of full 3D structure of magnetic field and core flow.
- Study the underlying physics.



The Physics of the Geodynamo

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$

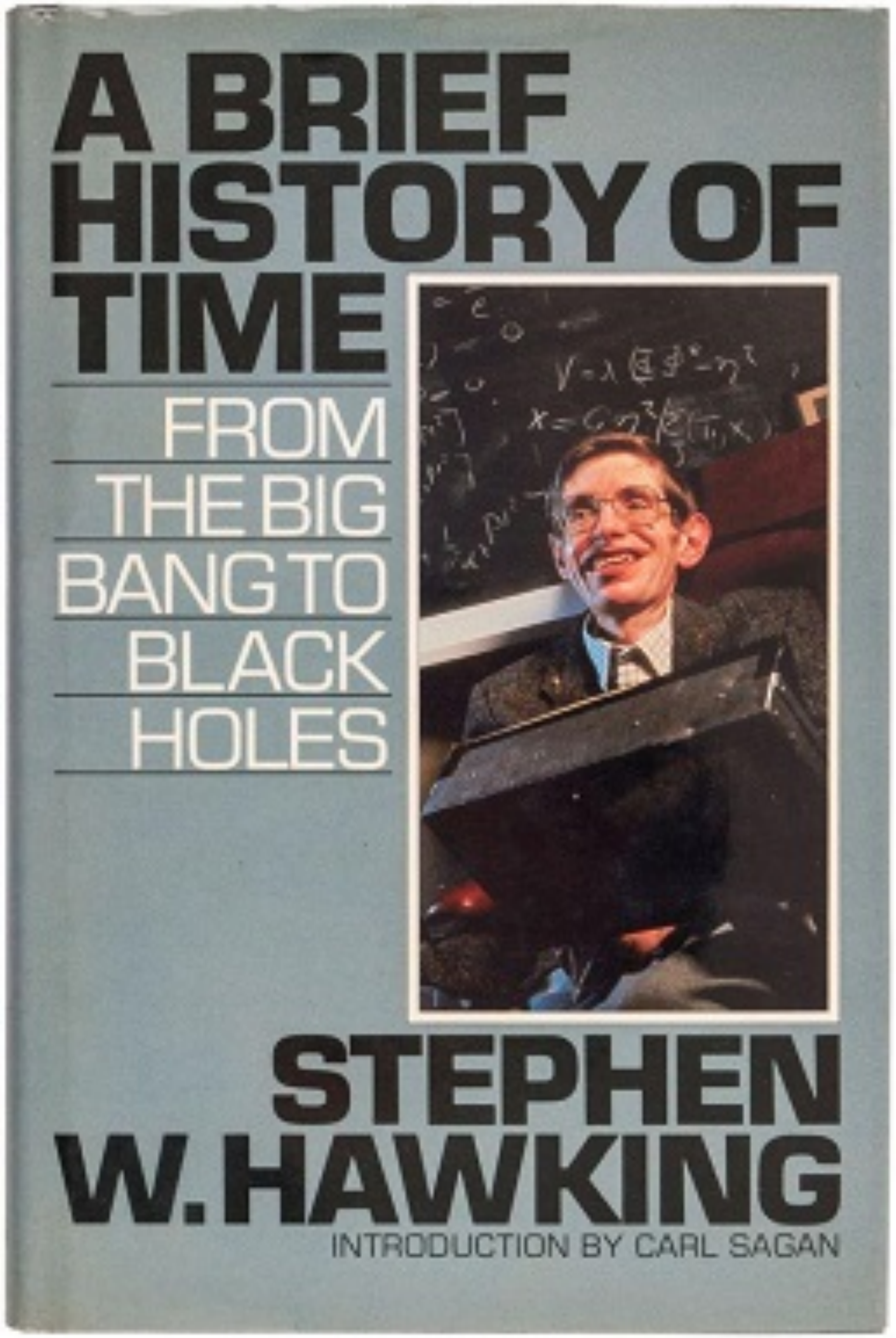
$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma \rho c_p}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad \rho = \rho_0 (1 - \alpha_v \delta T) \qquad \vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$

Some Good Advice (that I'm ignoring)

"Someone told me that each equation I included in the book would halve the sales. I therefore resolved not to have any equations at all. In the end, however, I did put in one equation, Einstein's famous equation, $E = mc^2$. I hope that this will not scare off half of my potential readers."



Solve the Dynamo Equations?

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$

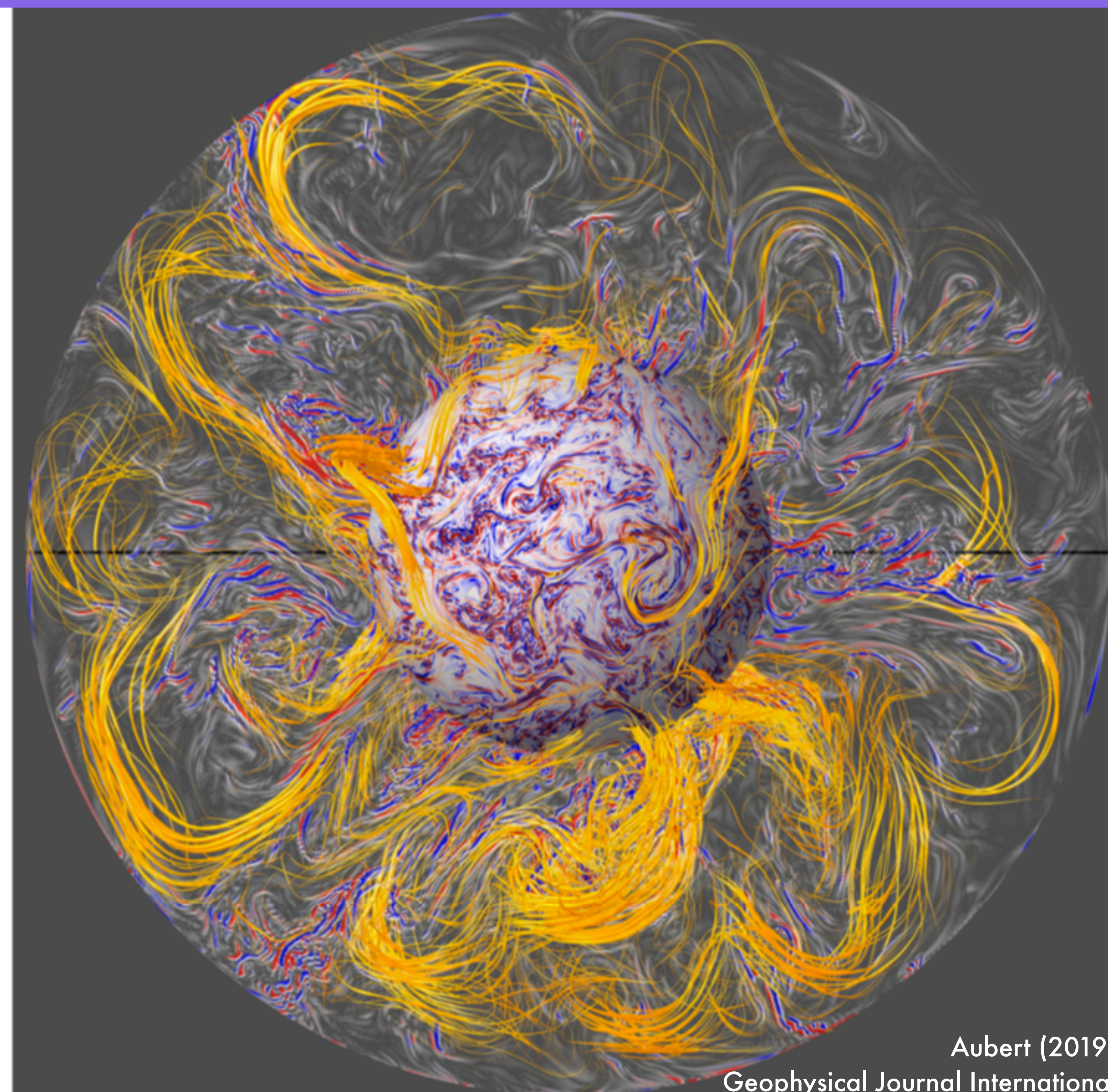
$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma \rho c_p}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad \rho = \rho_0(1 - \alpha_v \delta T) \qquad \vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$

Solve the Dynamo Equations?

- Computer simulations.
- (Tens of) Millions of CPU hours for a single simulation.
- Still only an approximation of the equations and the Earth.



Solve the Dynamo Equations?

- Clay Millennium Problems.
- US$1,000,000 prize for each.
- “The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.”

CMI

ABOUT PROGRAMS MILLENNIUM PROBLEMS PEOPLE PUBLICATIONS EVENTS EUCLID

Millennium Problems

Yang–Mills and Mass Gap
Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Riemann Hypothesis
The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2.

P vs NP Problem
If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? This is the essence of the P vs NP question. Typical of the NP problems is that of the Hamiltonian Path Problem: given N cities to visit, how can one do this without visiting a city twice? If you give me a solution, I can easily check that it is correct. But I cannot so easily find a solution.

Navier–Stokes Equation
This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

Hodge Conjecture
The answer to this conjecture determines how much of the topology of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations. The Hodge conjecture is known in certain special cases, e.g., when the solution set has dimension less than four. But in dimension four it is unknown.

Poincaré Conjecture
In 1904 the French mathematician Henri Poincaré asked if the three dimensional sphere is characterized as the unique simply connected three manifold. This question, the Poincaré conjecture, was a special case of Thurston's geometrization conjecture. Perelman's proof tells us that every three manifold is built from a set of standard pieces, each with one of eight well-understood geometries.

Birch and Swinnerton-Dyer Conjecture
Supported by much experimental evidence, this conjecture relates the number of points on an elliptic curve mod p to the rank of the group of rational points. Elliptic curves, defined by cubic equations in two variables, are fundamental mathematical objects that arise in many areas: Wiles' proof of the Fermat Conjecture, factorization of numbers into primes, and cryptography, to name three.

Solve the Dynamo Equations?

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma \rho c_p}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad \rho = \rho_0(1 - \alpha_v \delta T) \qquad \vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$

Understand the Equations?

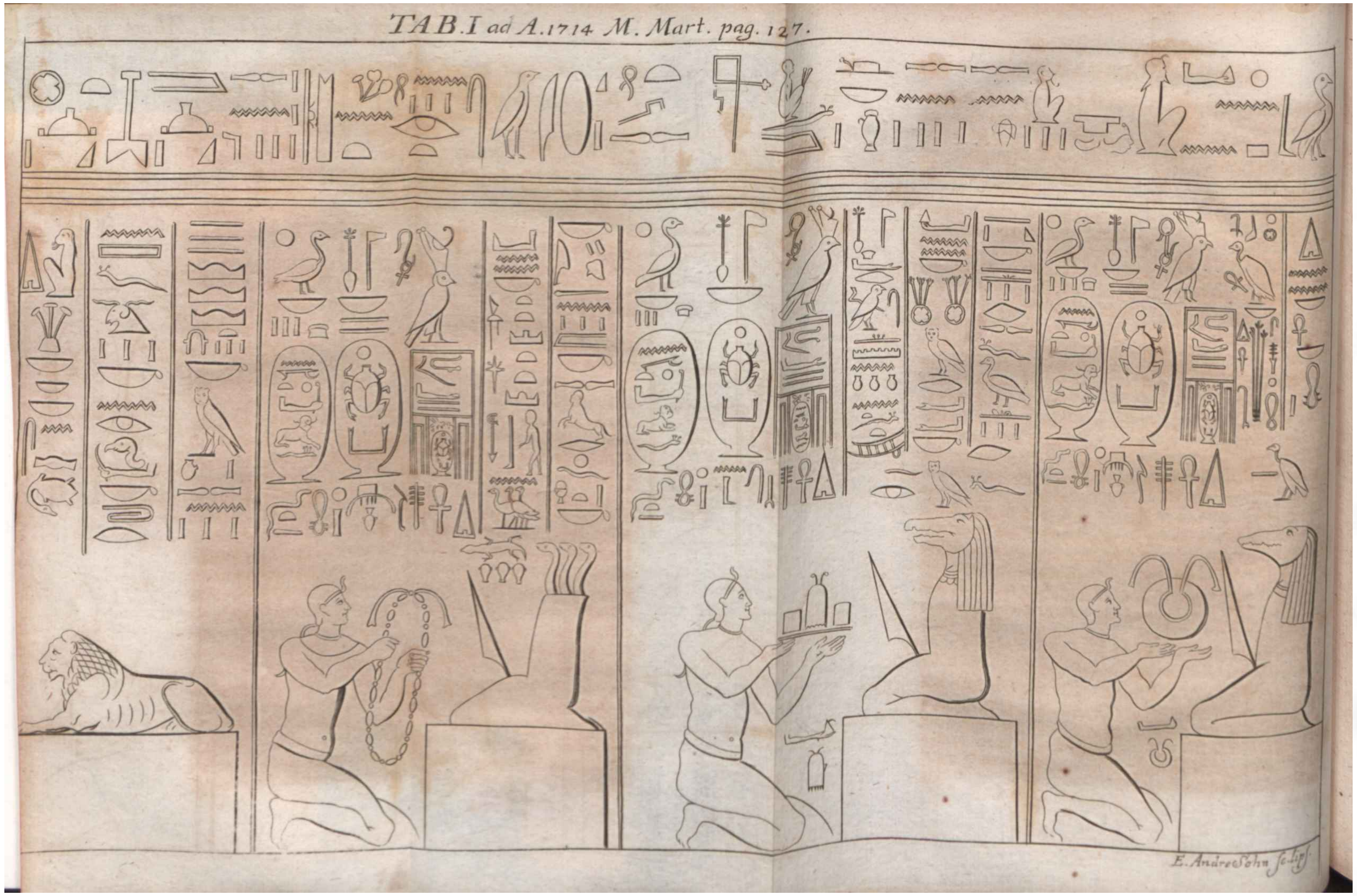
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma \rho c_p}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad \rho = \rho_0(1 - \alpha_v \delta T) \qquad \vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$

Understanding?

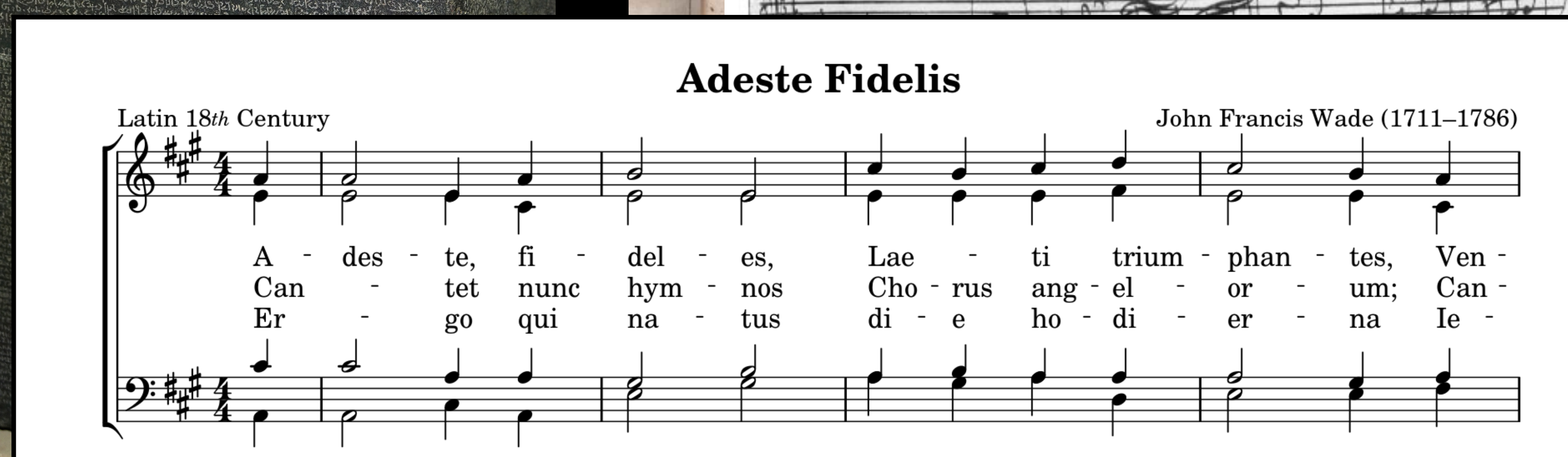


Images: wikipedia

Translation before Understanding

Images: wikipedia

Simple before Complex



Images: wikipedia

Translate and Understand Simplified Equations?

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma \rho c_p}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad \rho = \rho_0 (1 - \alpha_v \delta T) \qquad \vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$

Inside the Earth

- Thin rocky crust, generally 5-50 km thick
- Thick rocky mantle, 2900 km thick
- Liquid metal outer core 2260 km thick
- Solid metal inner core 1220 km radius

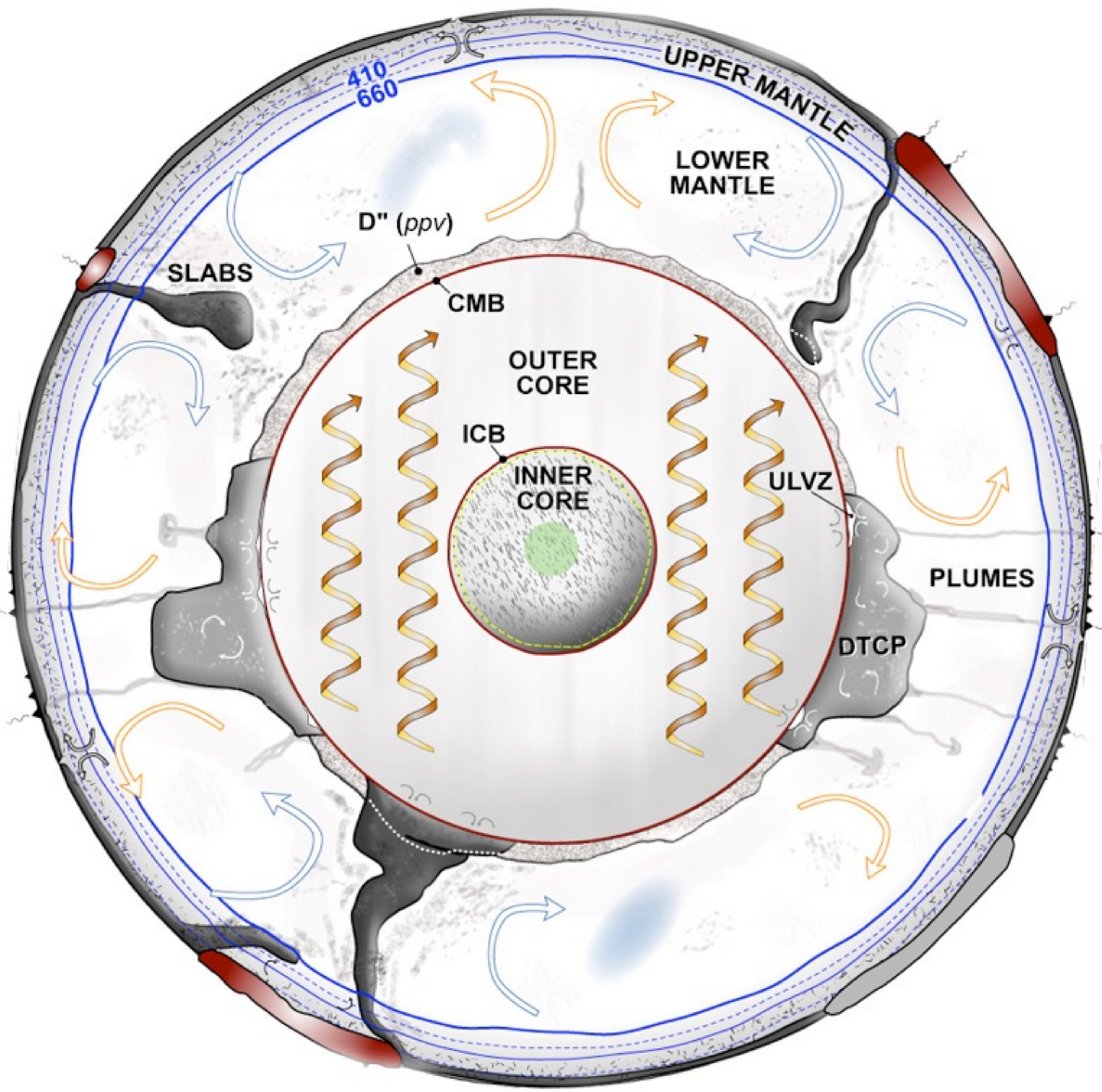


Image: Ed Garnero, Arizona State University

Understand Simplified Equations?

- Acceleration due to forces.
- Energy conservation: “sources” and “loss”.
- Magnetic fields, electric currents, motion.

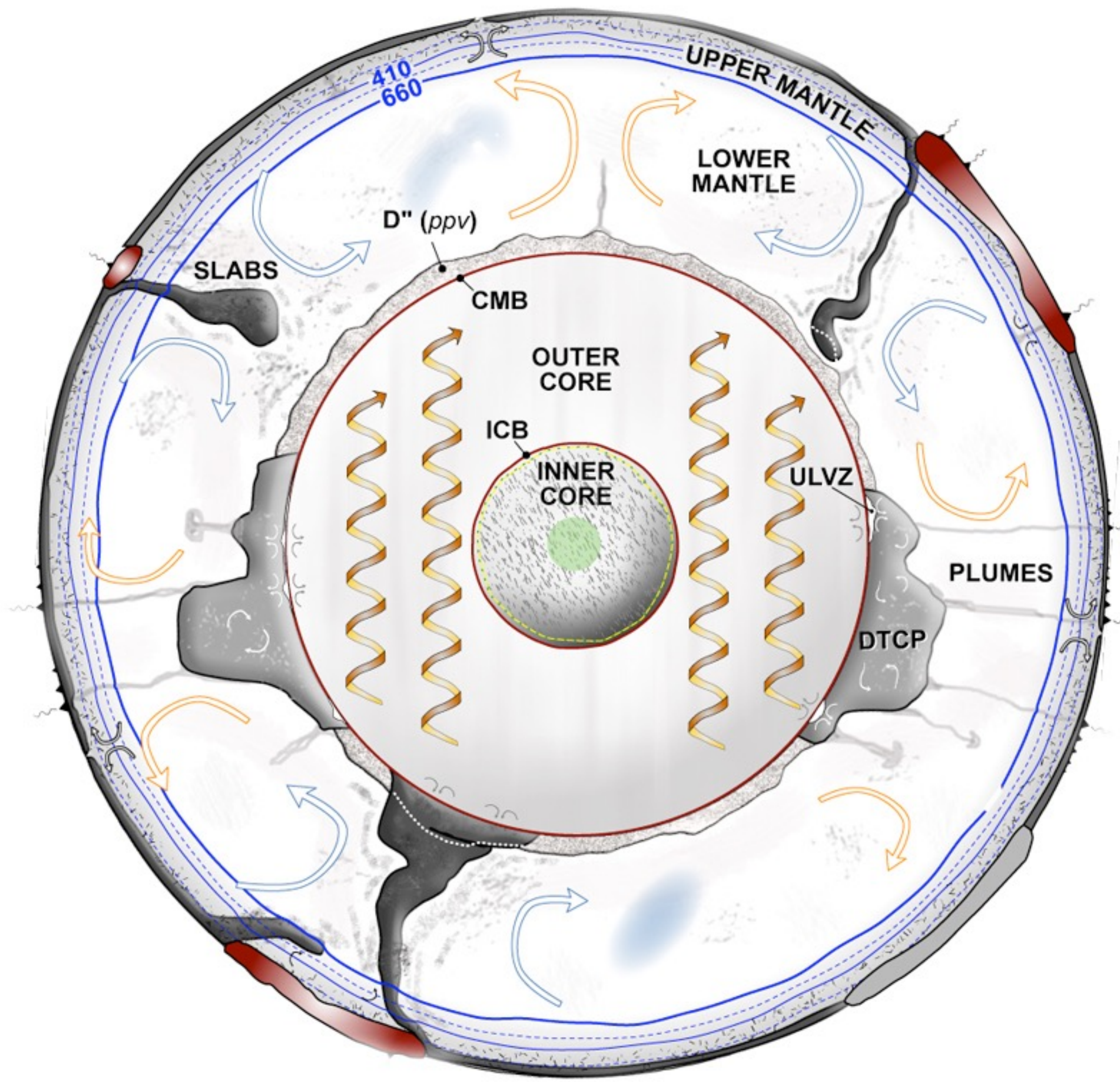
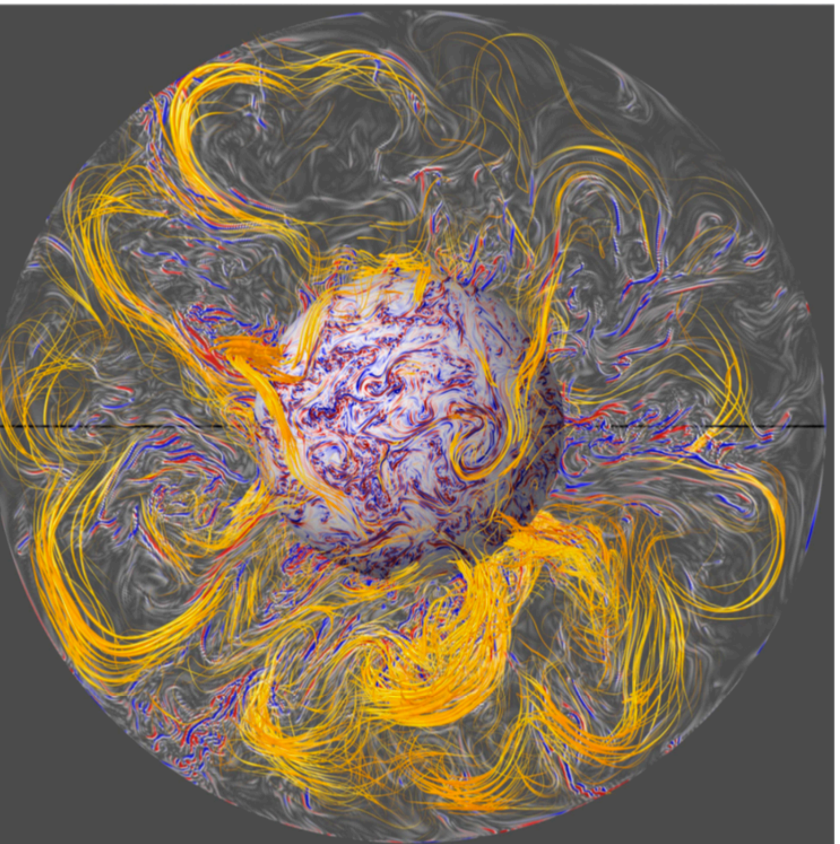


Image: Ed Garnero, Arizona State University

Acceleration due to Forces

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$



Aubert (2019)
Geophysical Journal International

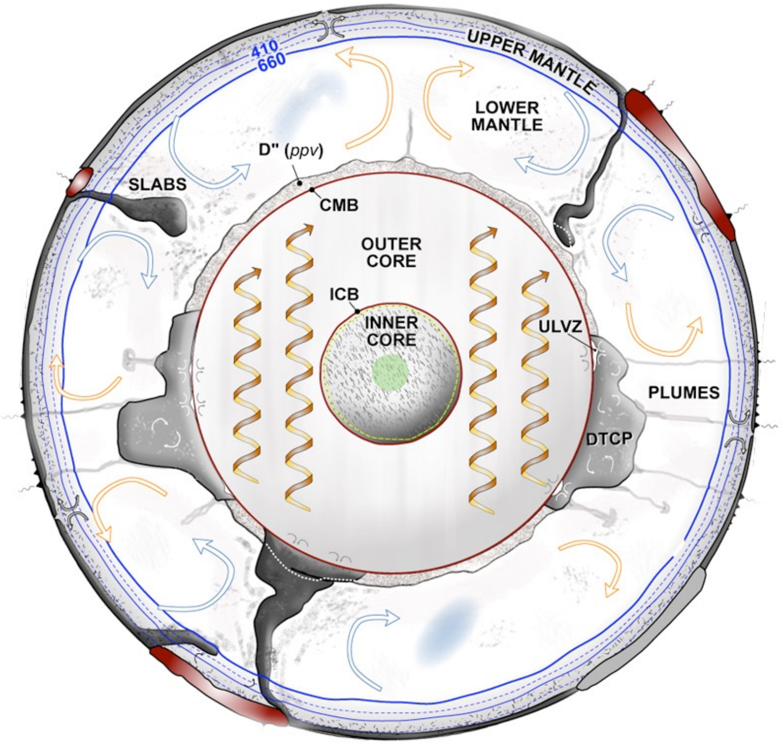
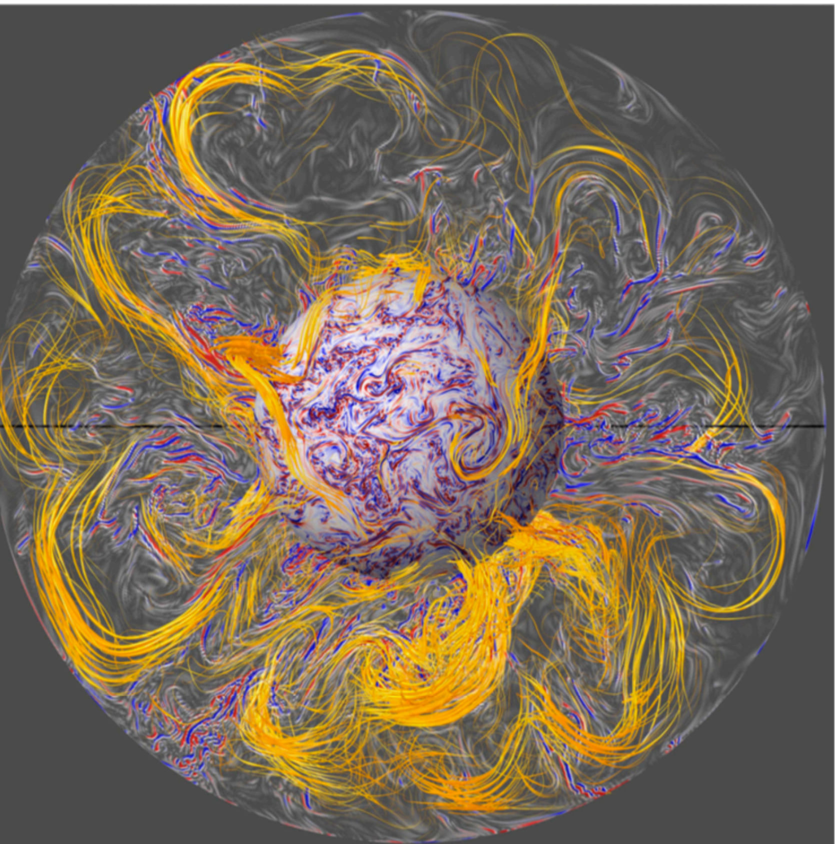


Image: Ed Garnero, Arizona State University

Acceleration due to Forces

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$

Acceleration



Aubert (2019)
Geophysical Journal International

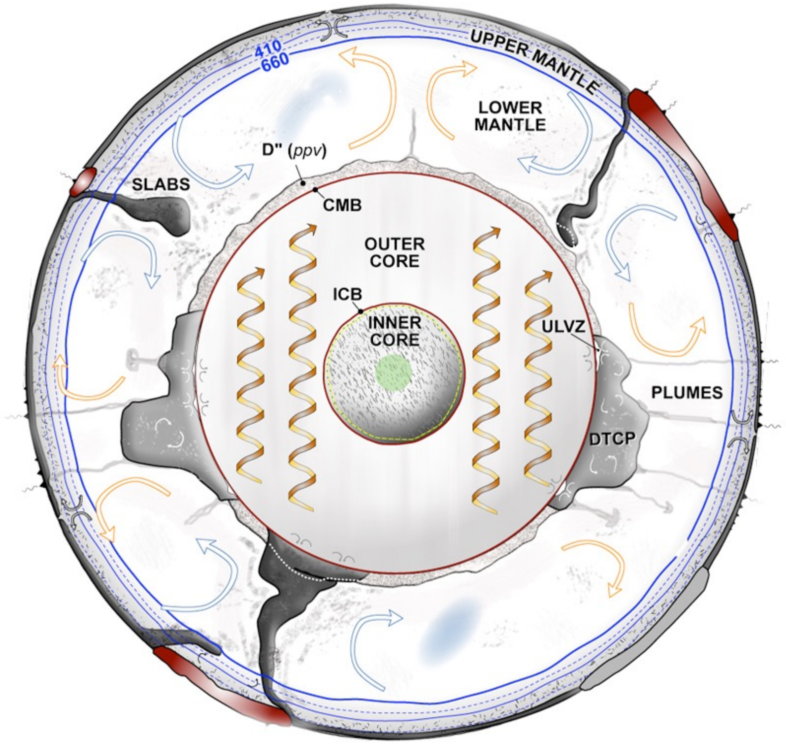


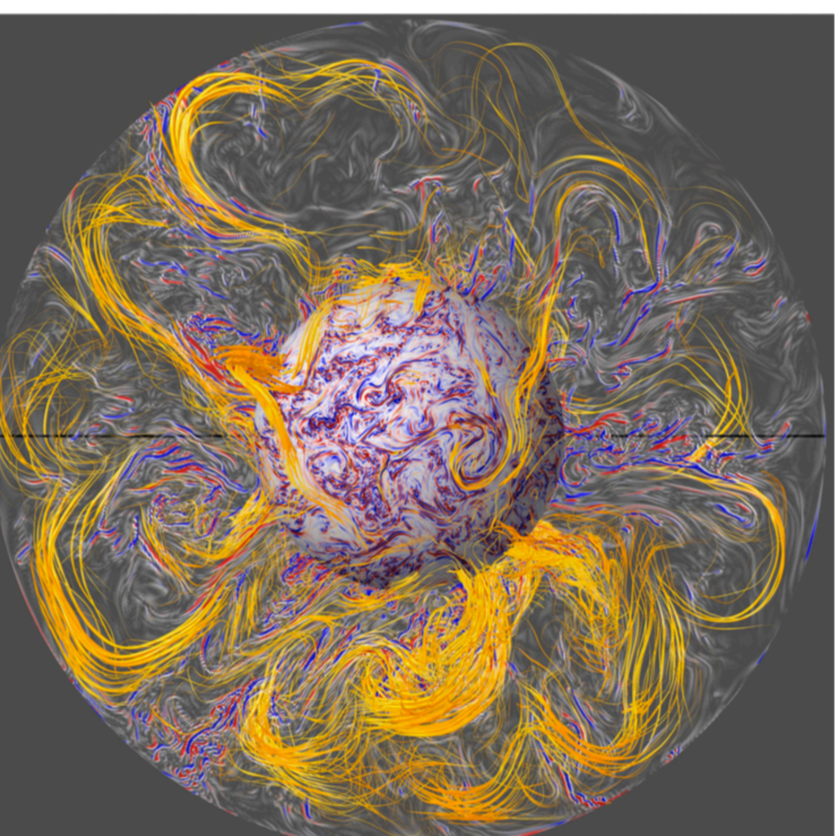
Image: Ed Garnero, Arizona State University

Acceleration due to Forces

Gravity

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$

Acceleration



Aubert (2019)
Geophysical Journal International

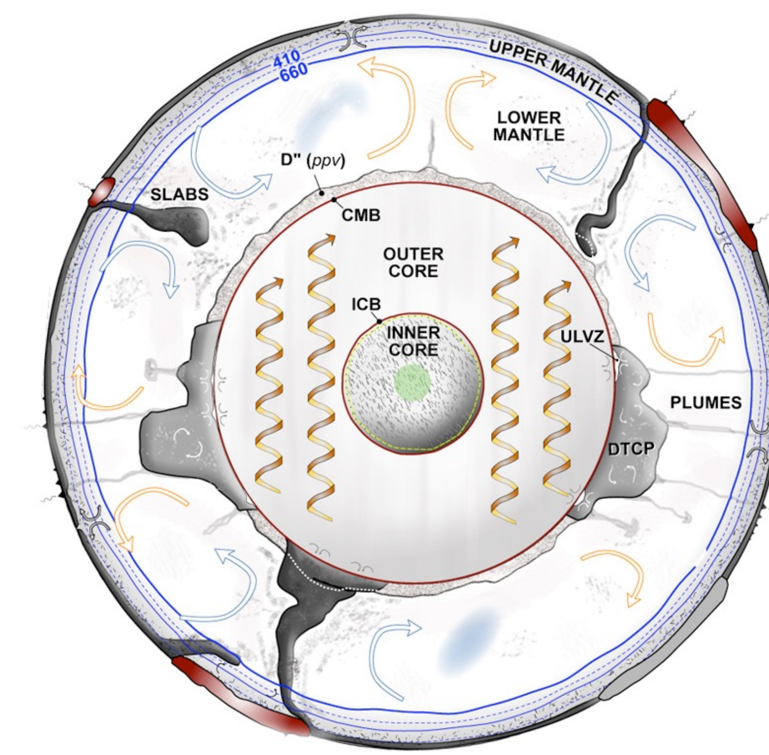


Image: Ed Garnero, Arizona State University

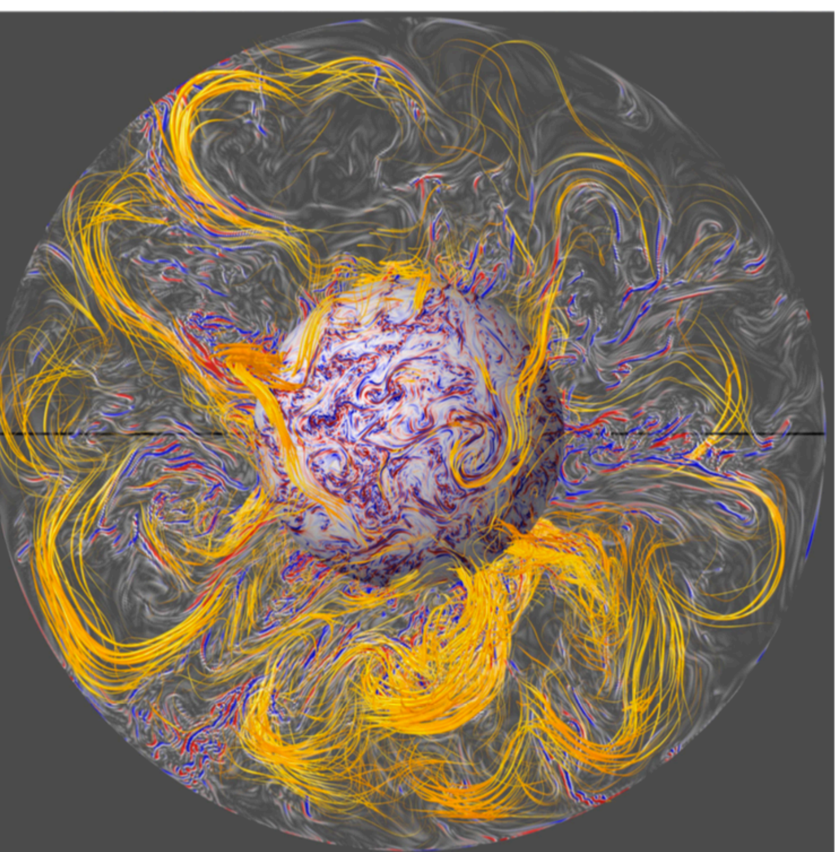
Acceleration due to Forces

Gravity

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$

Acceleration

Magnetic Force



Aubert (2019)
Geophysical Journal International

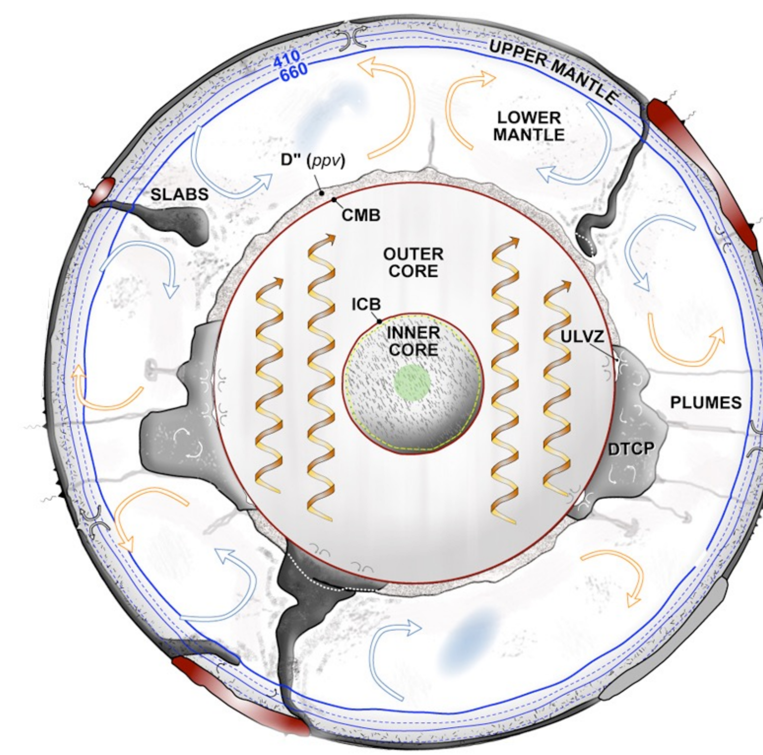


Image: Ed Garnero, Arizona State University

Acceleration due to Forces

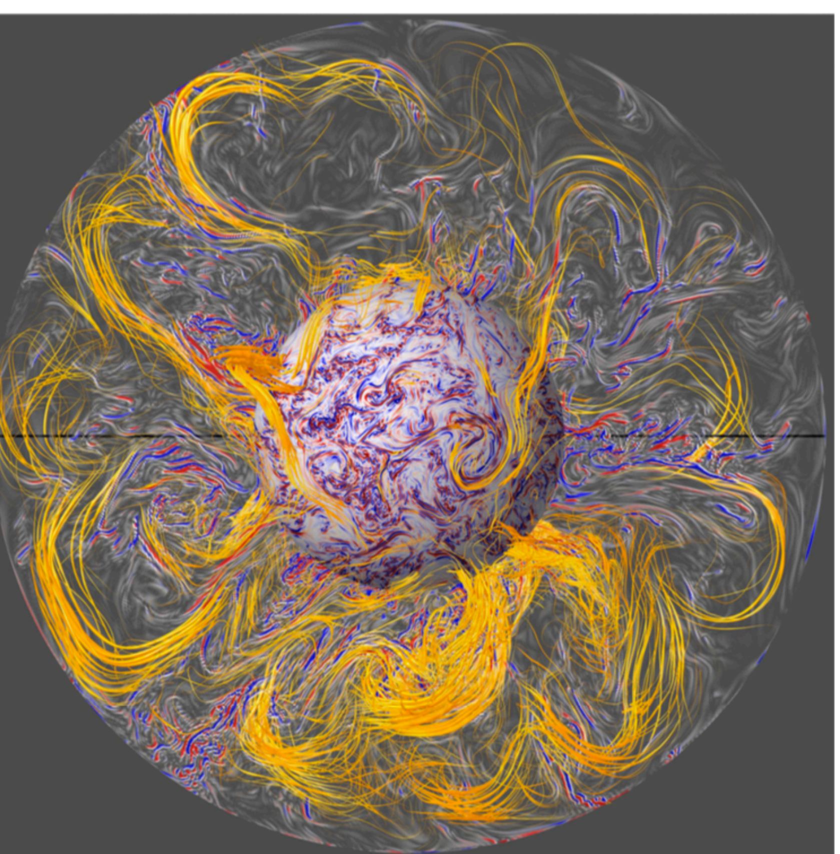
Gravity

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)(\vec{v}) + 2\vec{\Omega}\times\vec{v} = -\nabla\Pi + \frac{1}{\rho_0}(\vec{j}\times\vec{B}) + \alpha_v \delta T \vec{g} + \nu\nabla^2\vec{v}$$

Acceleration

Magnetic Force

Drag



Aubert (2019)
Geophysical Journal International

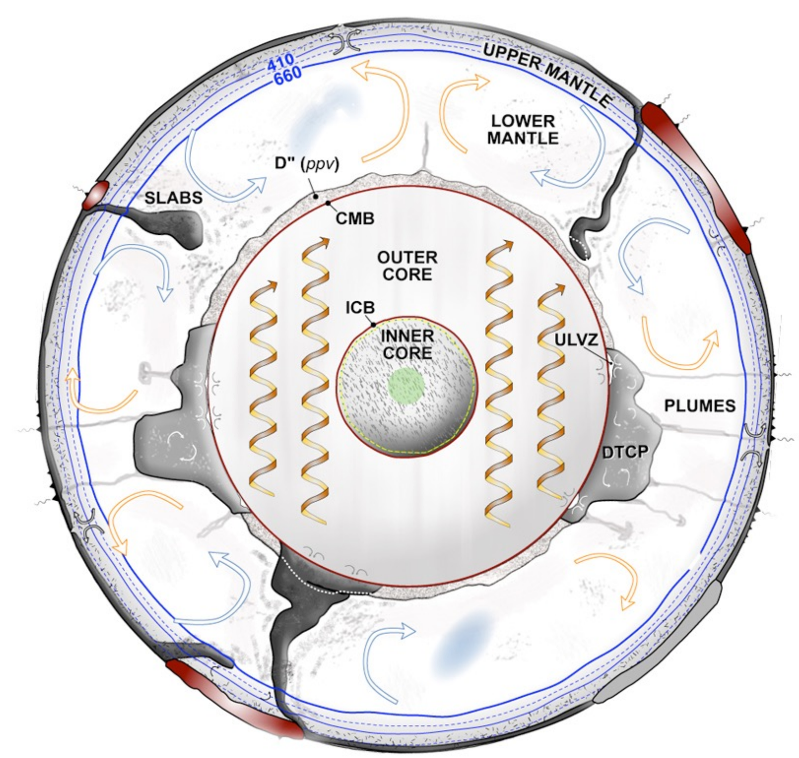


Image: Ed Garnero, Arizona State University

Acceleration due to Forces

Gravity

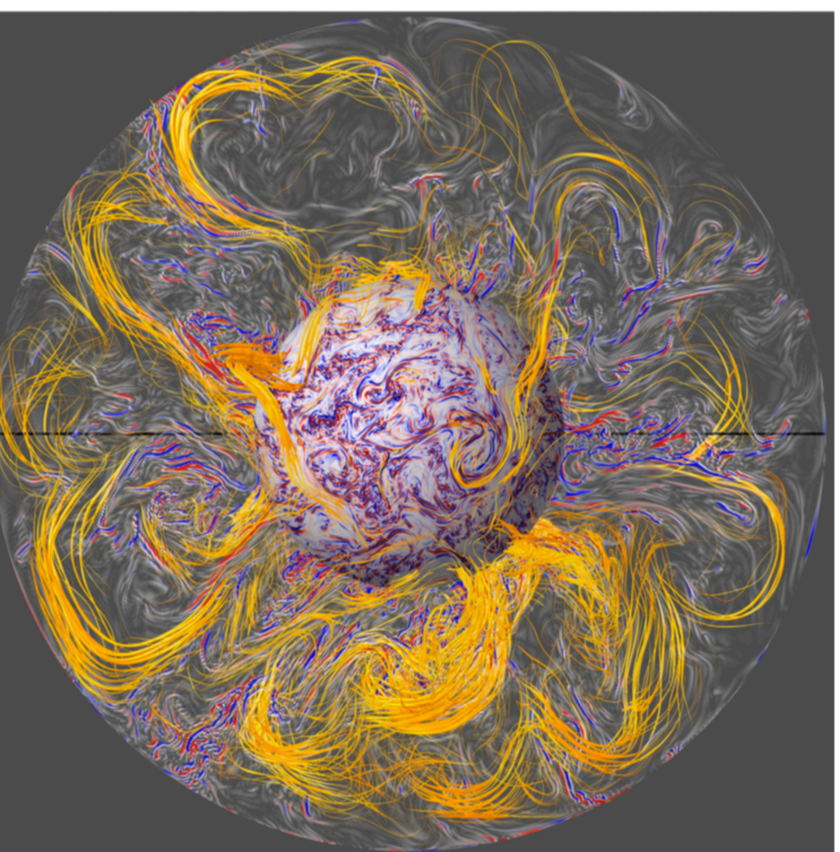
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)(\vec{v}) + 2\vec{\Omega}\times\vec{v} = -\nabla\Pi + \frac{1}{\rho_0}(\vec{j}\times\vec{B}) + \alpha_v \delta T \vec{g} + \nu\nabla^2\vec{v}$$

Acceleration

Rotation

Magnetic Force

Drag



Aubert (2019)
Geophysical Journal International

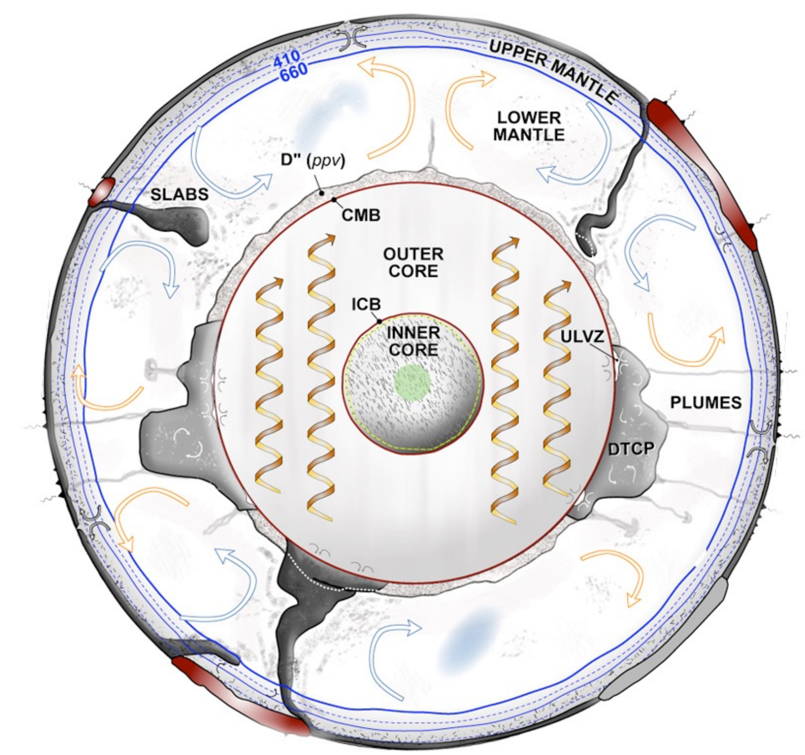


Image: Ed Garnero, Arizona State University

Acceleration due to Forces

Gravity

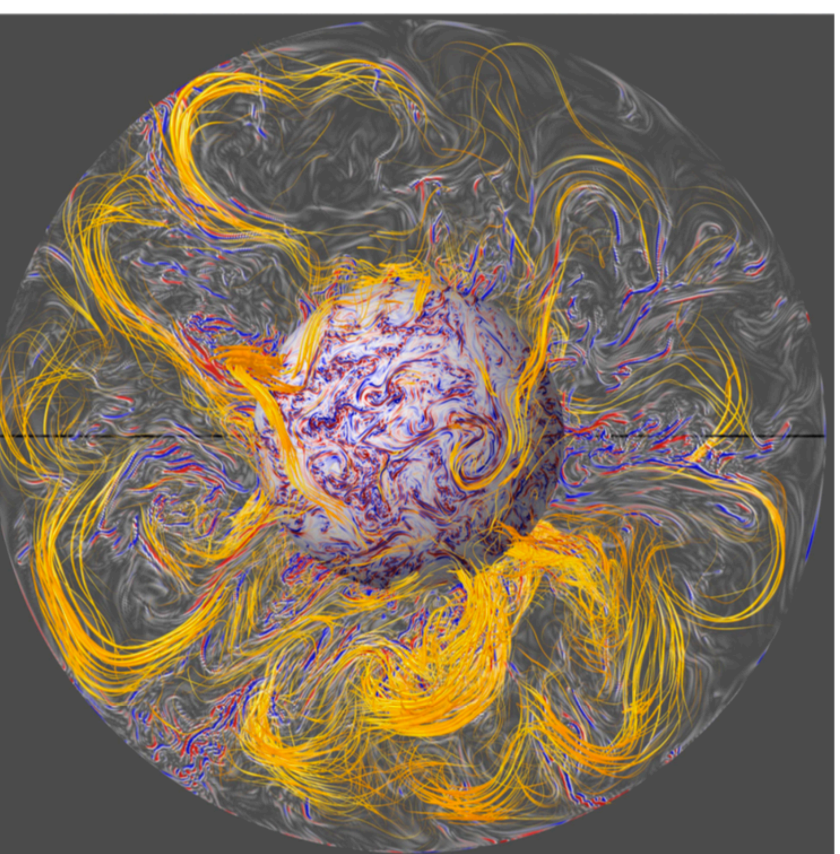
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)(\vec{v}) + 2\vec{\Omega}\times\vec{v} = -\nabla\Pi + \frac{1}{\rho_0}(\vec{j}\times\vec{B}) + \alpha_v \delta T \vec{g} + \nu\nabla^2\vec{v}$$

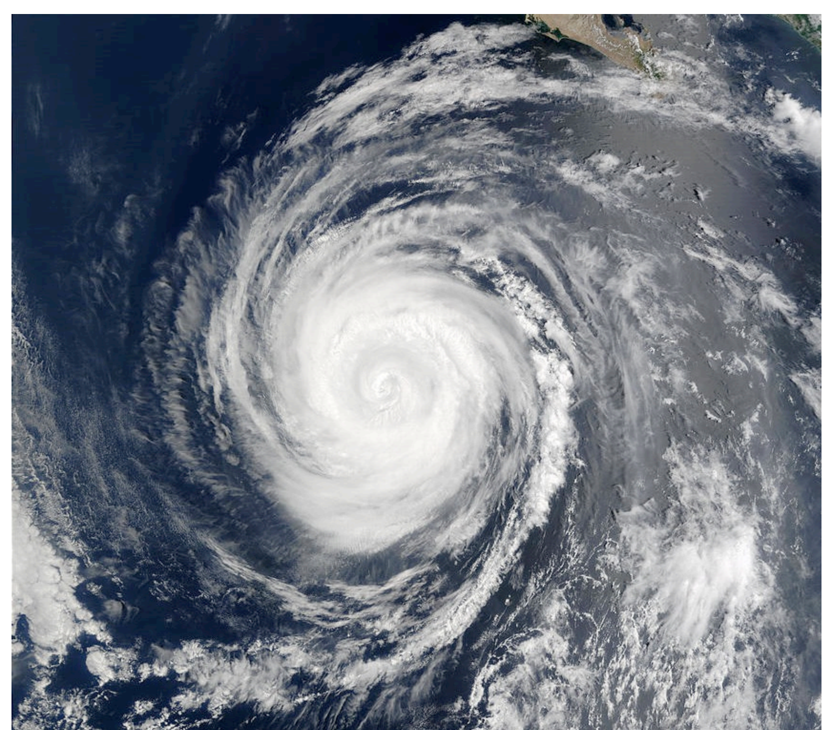
Acceleration

Rotation

Magnetic Force

Drag





Aubert (2019)
Geophysical Journal International

visibleearth.nasa.gov

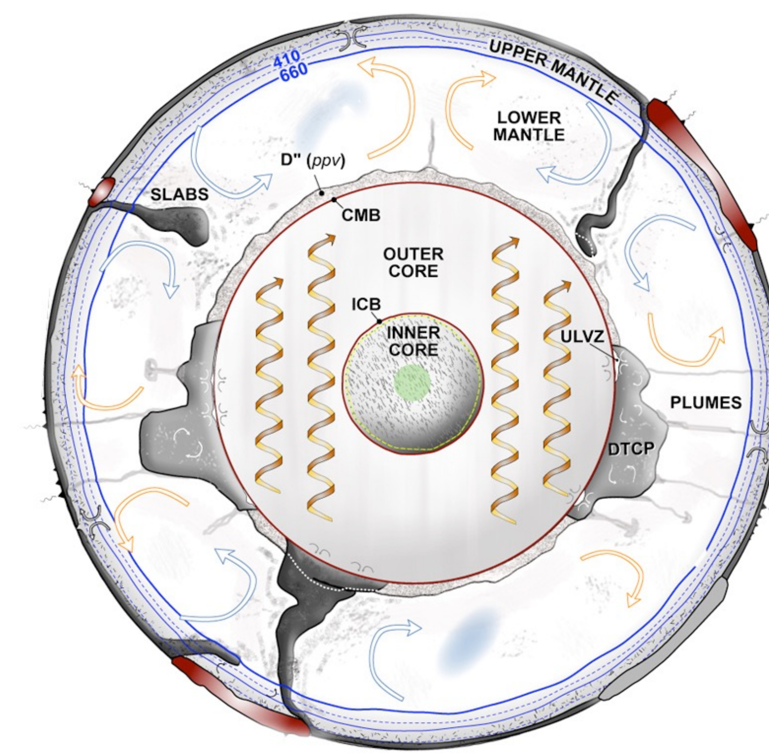
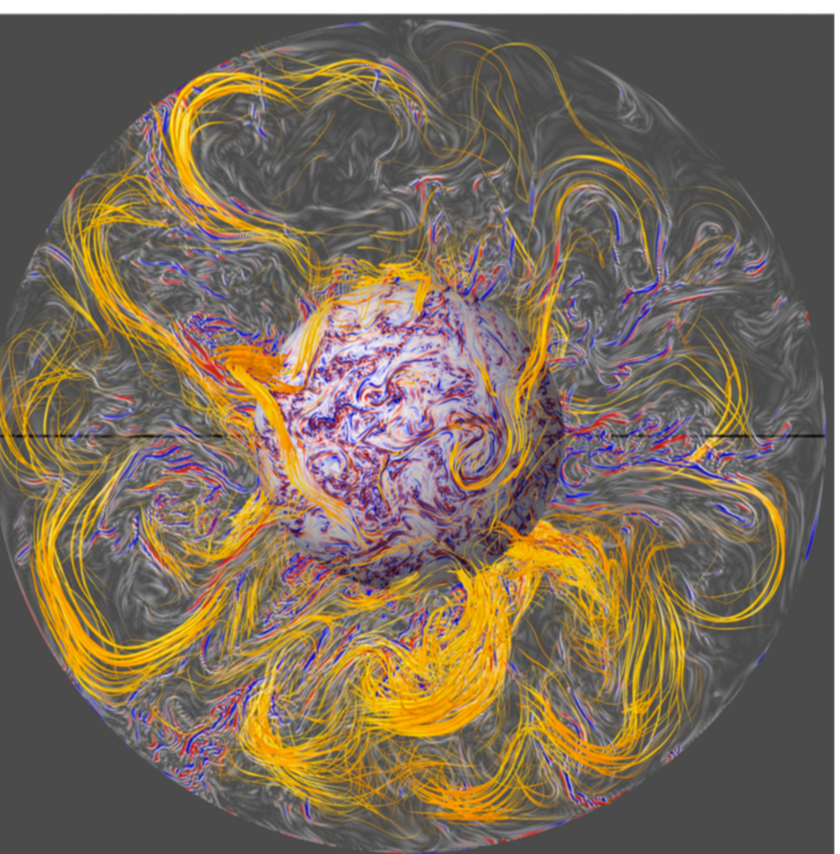
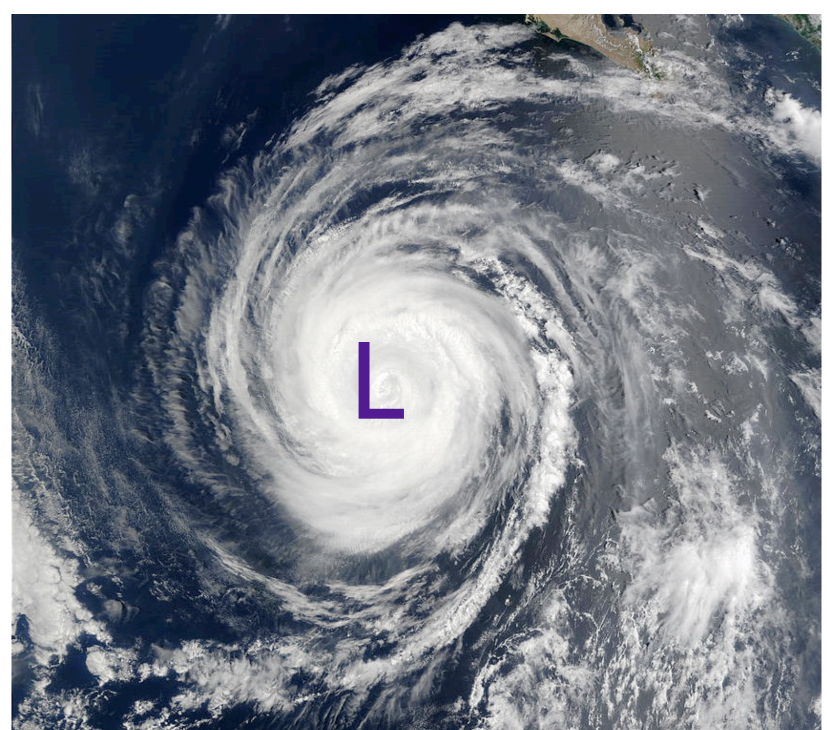


Image: Ed Garnero, Arizona State University

Acceleration due to Forces

$$\underbrace{\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)(\vec{v})}_{\text{Acceleration}} + \underbrace{2\vec{\Omega}\times\vec{v}}_{\text{Rotation}} = \overbrace{-\nabla\Pi}^{\text{Pressure}} + \underbrace{\frac{1}{\rho_0}(\vec{j}\times\vec{B})}_{\text{Magnetic Force}} + \overbrace{\alpha_v\delta T\vec{g}}^{\text{Gravity}} + \underbrace{\nu\nabla^2\vec{v}}_{\text{Drag}}$$





Aubert (2019)
Geophysical Journal International

visibleearth.nasa.gov

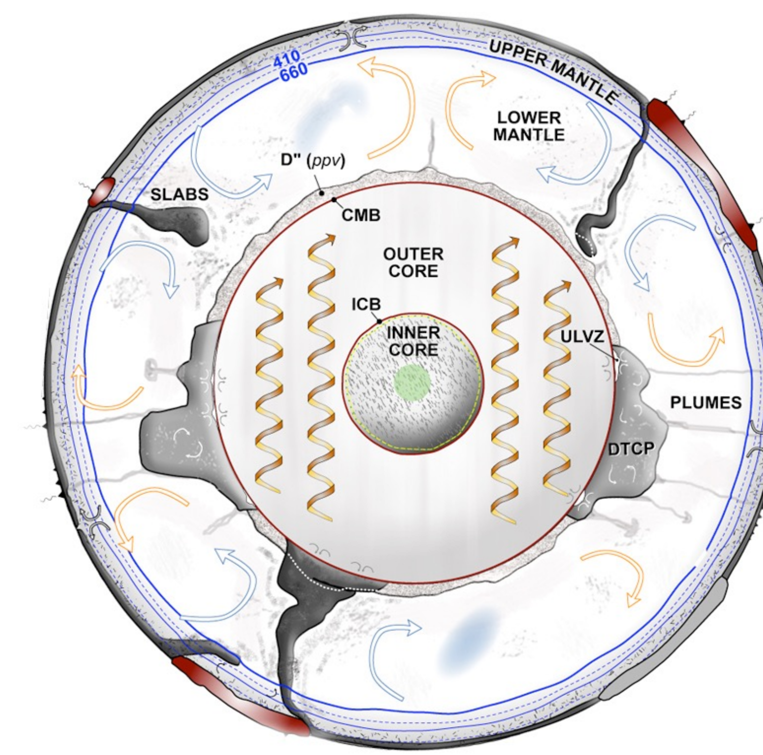
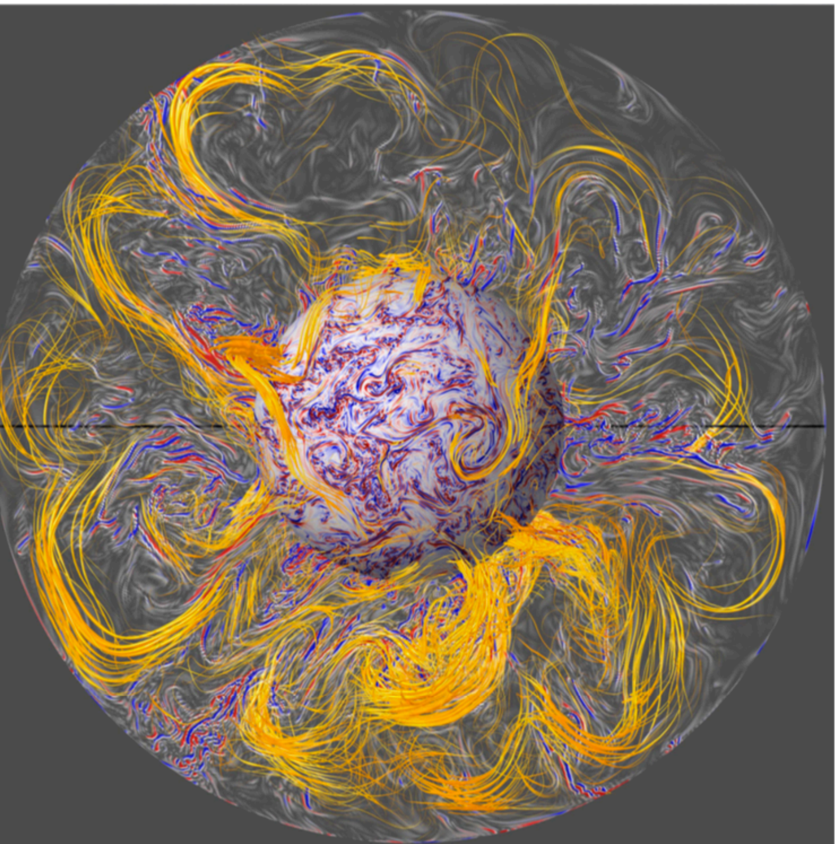


Image: Ed Garnero, Arizona State University

Energy Conservation: Sources & Loss

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma \rho c_p}$$



A dynamo is a system that generates a magnetic field, converting kinetic energy to magnetic energy.

Aubert (2019)
Geophysical Journal International

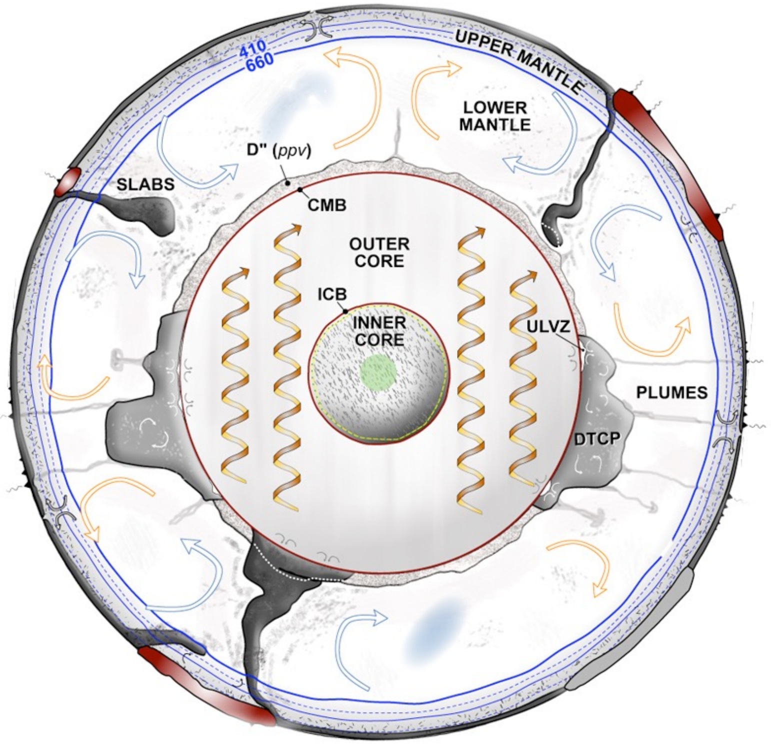
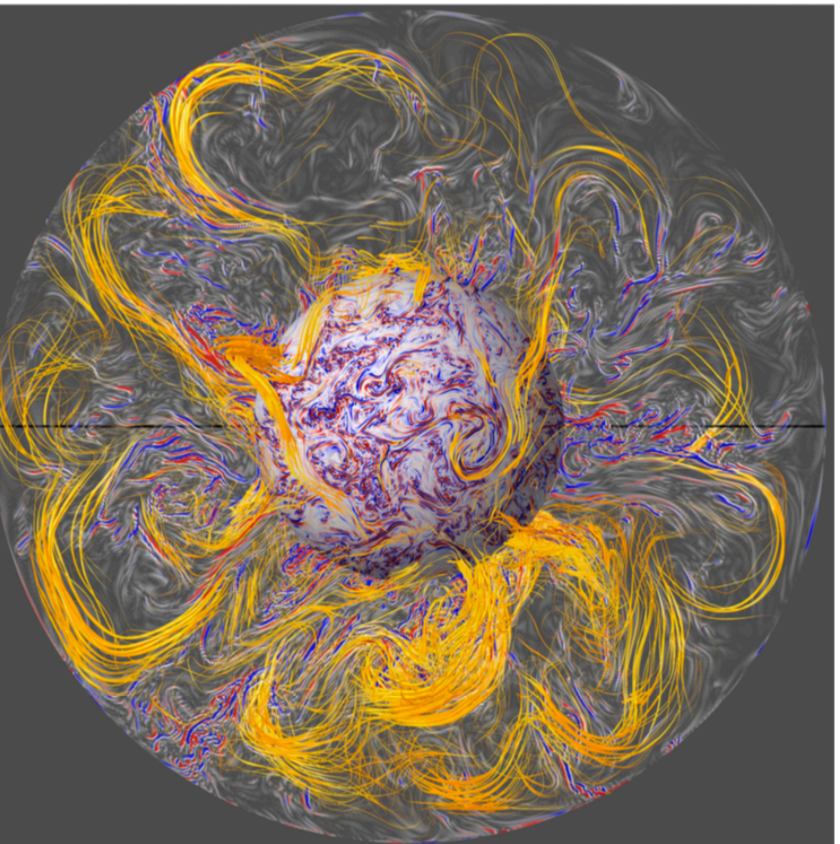


Image: Ed Garnero, Arizona State University

Energy Conservation: Sources & Loss

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma \rho c_p}$$

Cooling of the Core



A dynamo is a system that generates a magnetic field, converting kinetic energy to magnetic energy.

Aubert (2019)
Geophysical Journal International

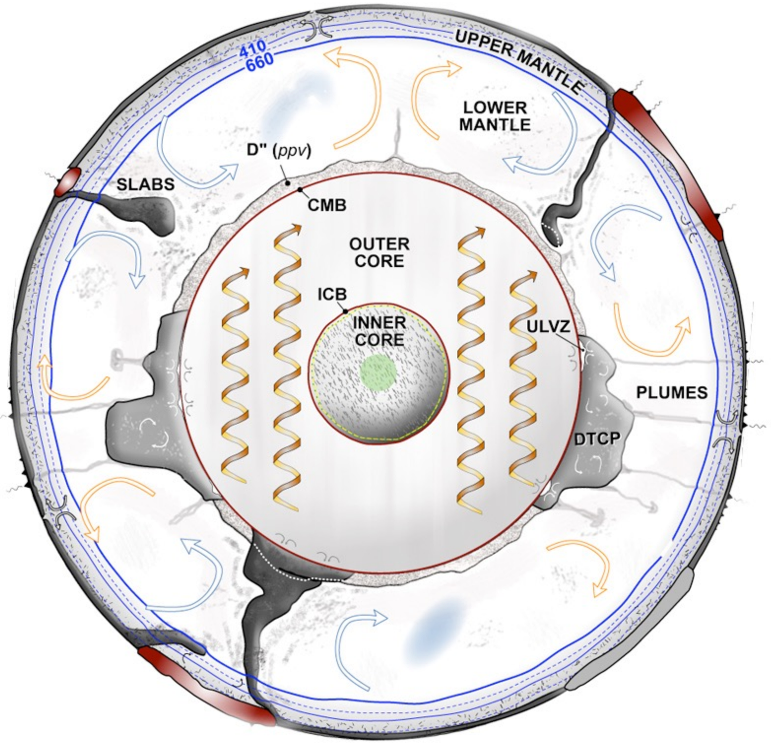


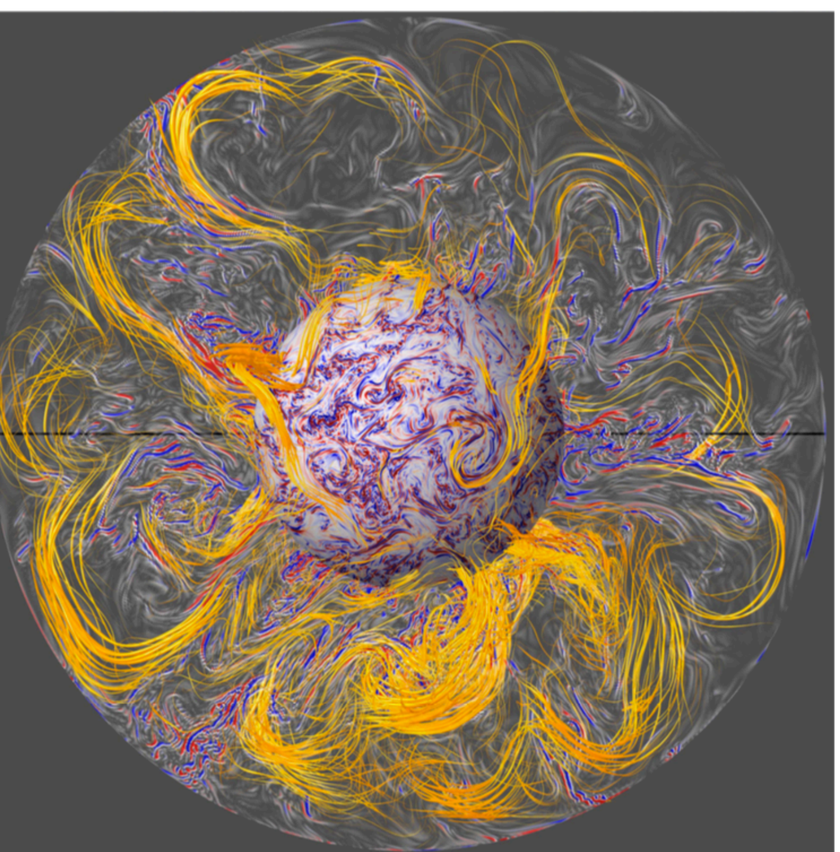
Image: Ed Garnero, Arizona State University

Energy Conservation: Sources & Loss

$$\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T = \kappa \nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma\,\rho c_p}$$

Cooling of the Core

Electrical Resistance



A dynamo is a system that generates a magnetic field, converting kinetic energy to magnetic energy.

Aubert (2019)
Geophysical Journal International

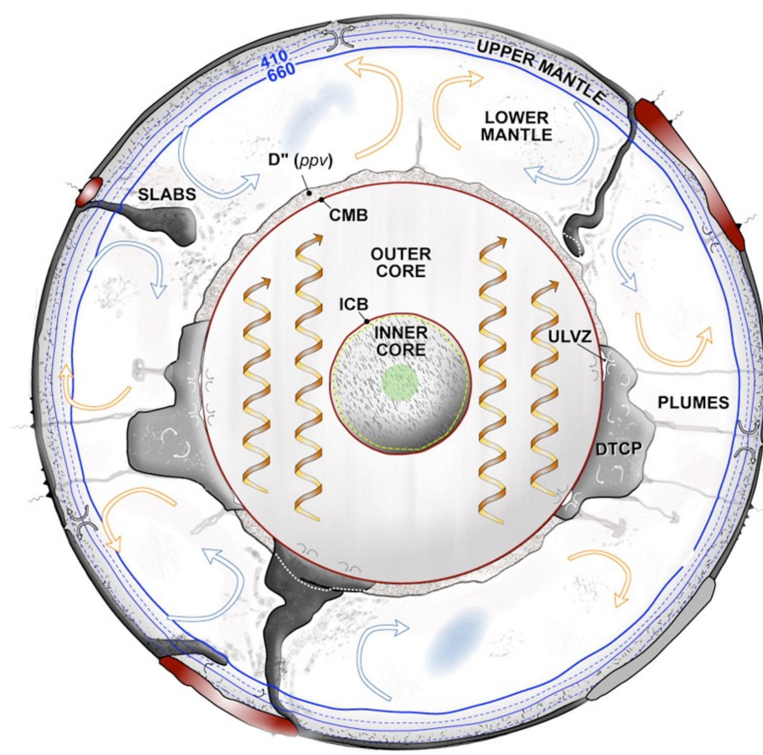


Image: Ed Garnero, Arizona State University

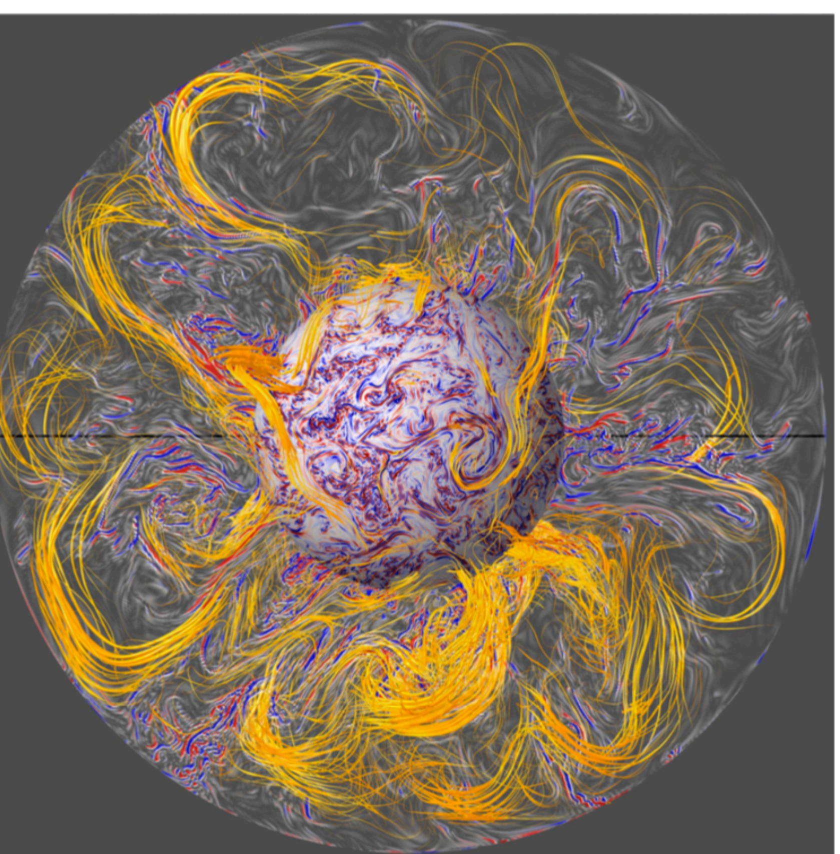
Energy Conservation: Sources & Loss

$$\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T = \kappa\nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma\,\rho c_p}$$

Cooling of the Core

Thermal Dissipation

Electrical Resistance



Aubert (2019)
Geophysical Journal International

A dynamo is a system that generates a magnetic field, converting kinetic energy to magnetic energy.

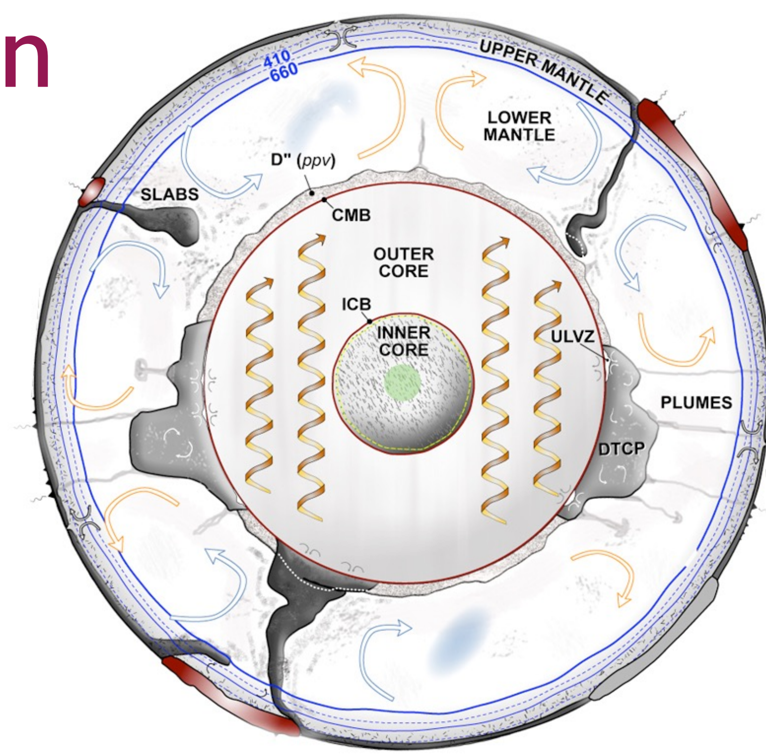


Image: Ed Garnero, Arizona State University

Energy Conservation: Sources & Loss

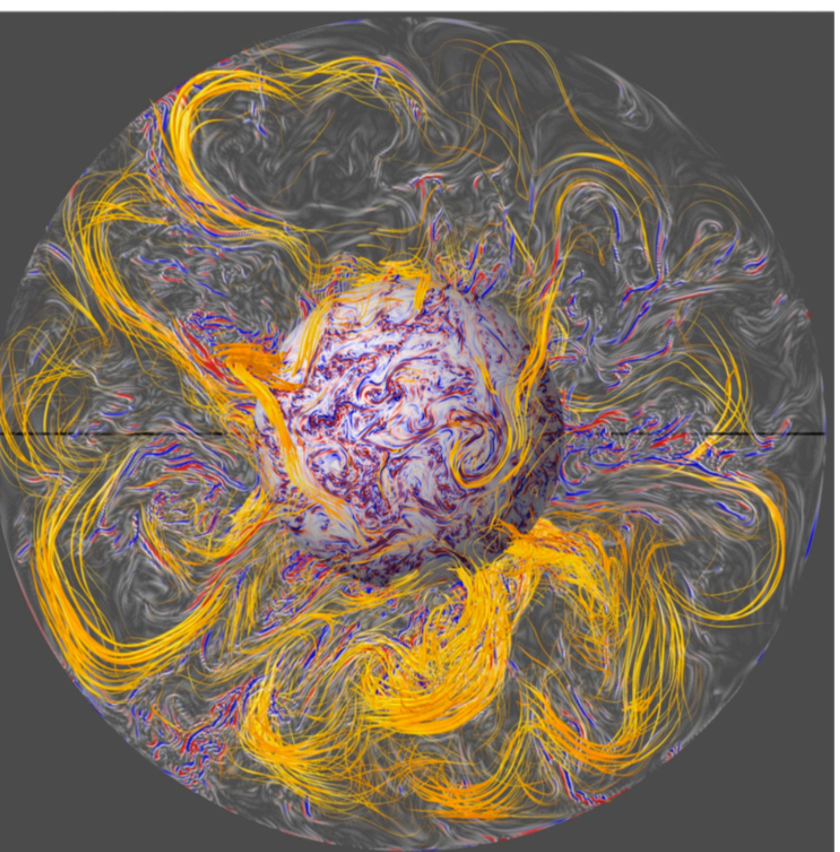
Radioactivity

$$\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T = \kappa\nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma\,\rho c_p}$$

Cooling of the Core

Thermal Dissipation

Electrical Resistance



A dynamo is a system that generates a magnetic field, converting kinetic energy to magnetic energy.

Aubert (2019)
Geophysical Journal International

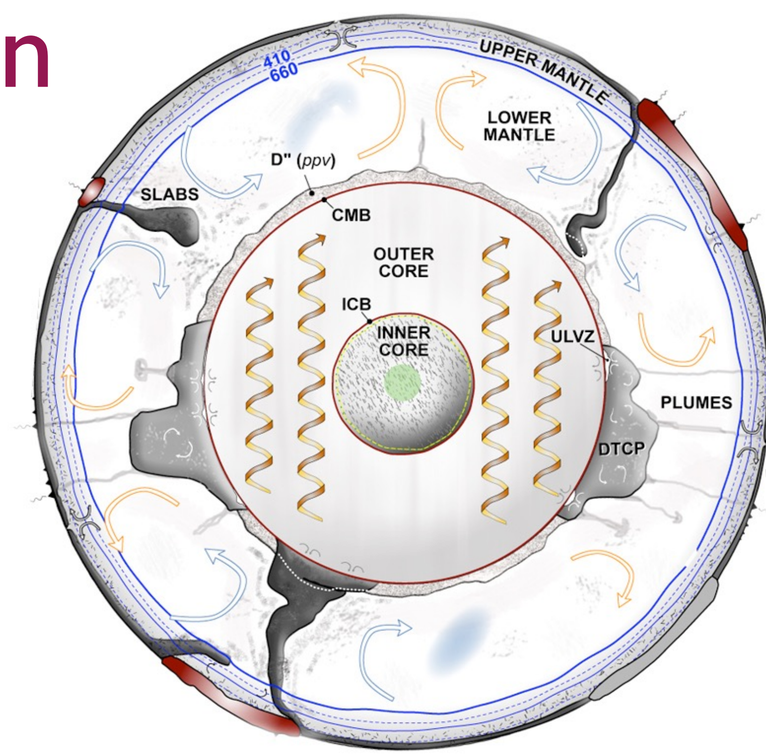
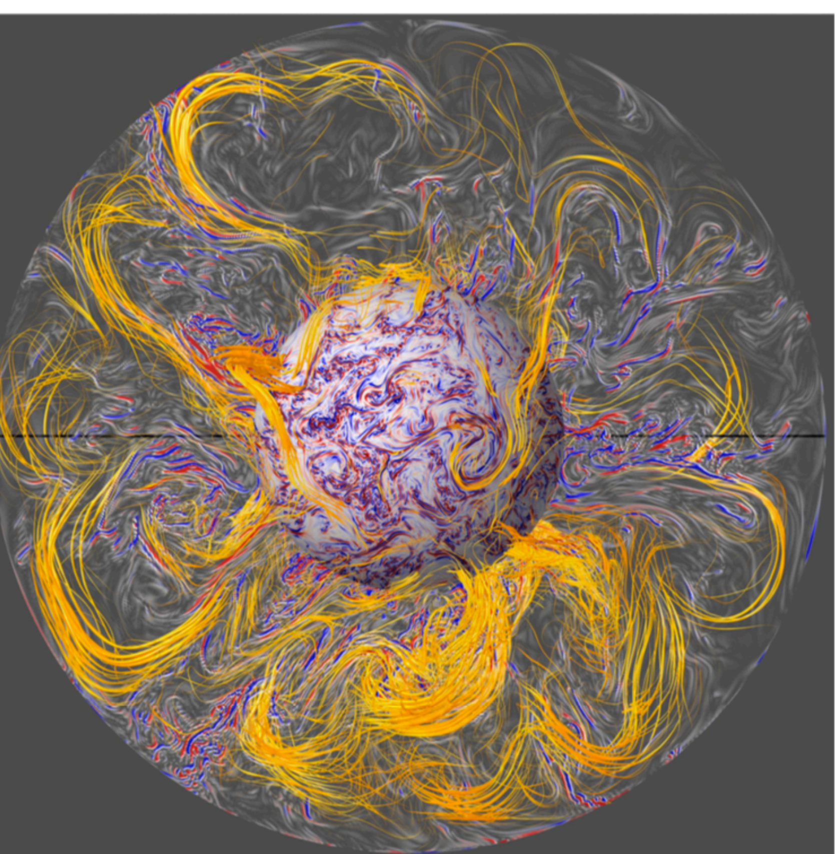


Image: Ed Garnero, Arizona State University

Magnetic Fields, Electric Currents, Motion

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$



Aubert (2019)
Geophysical Journal International

A dynamo is a system that generates a magnetic field, converting kinetic energy to magnetic energy

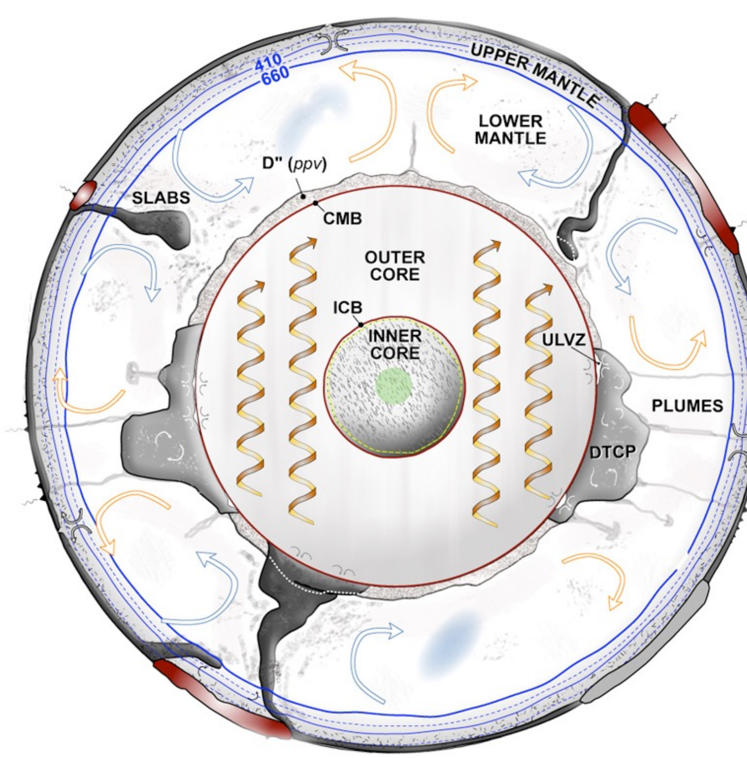
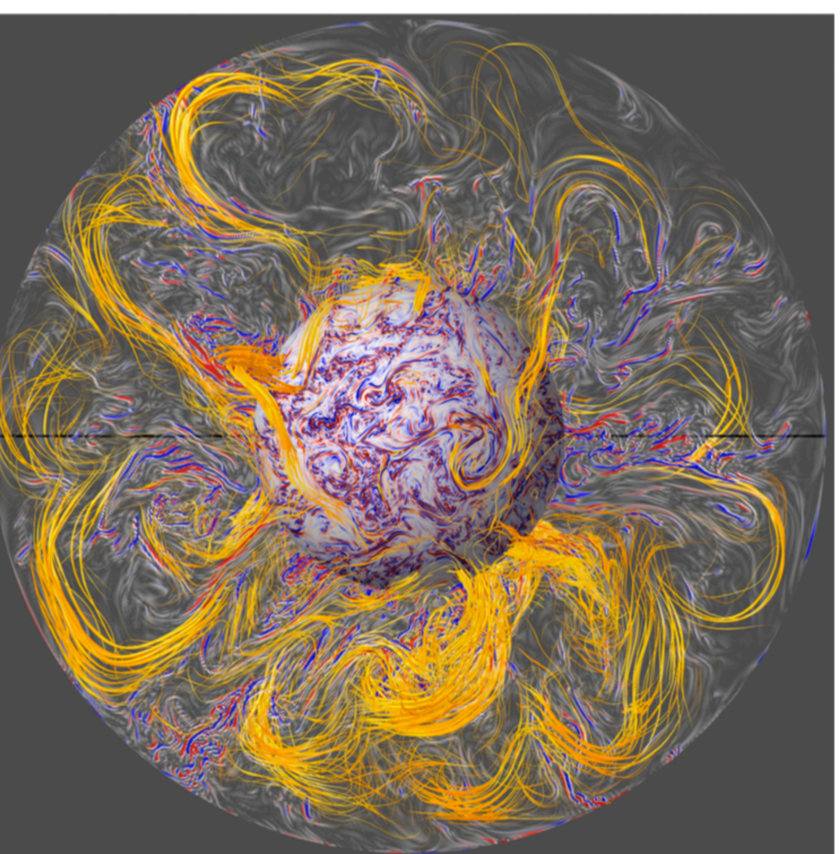


Image: Ed Garnero, Arizona State University

Magnetic Fields, Electric Currents, Motion

Magnetic Fields

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B} \qquad \vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$



A dynamo is a system that generates a magnetic field, converting kinetic energy to magnetic energy

Aubert (2019)
Geophysical Journal International

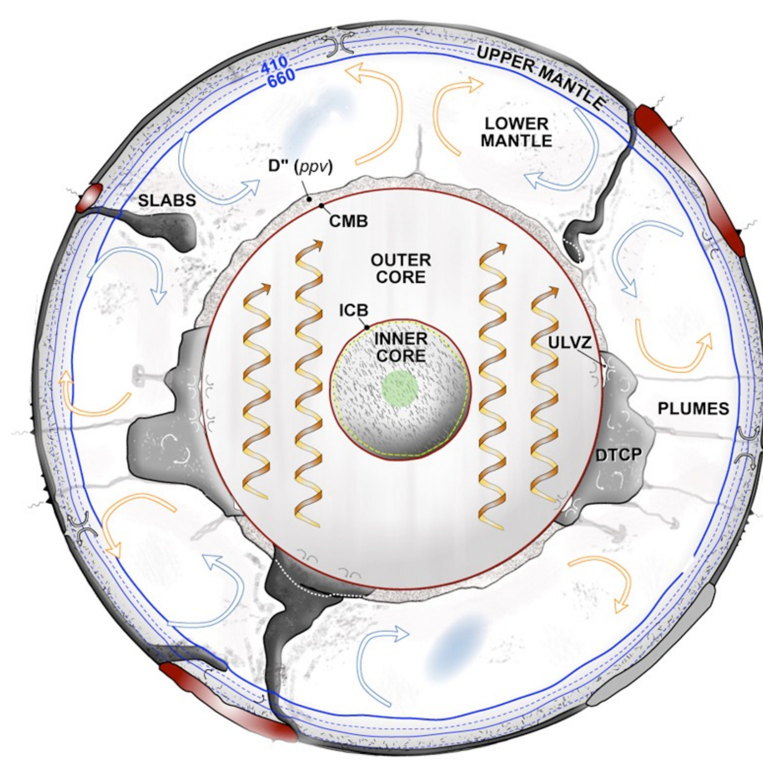


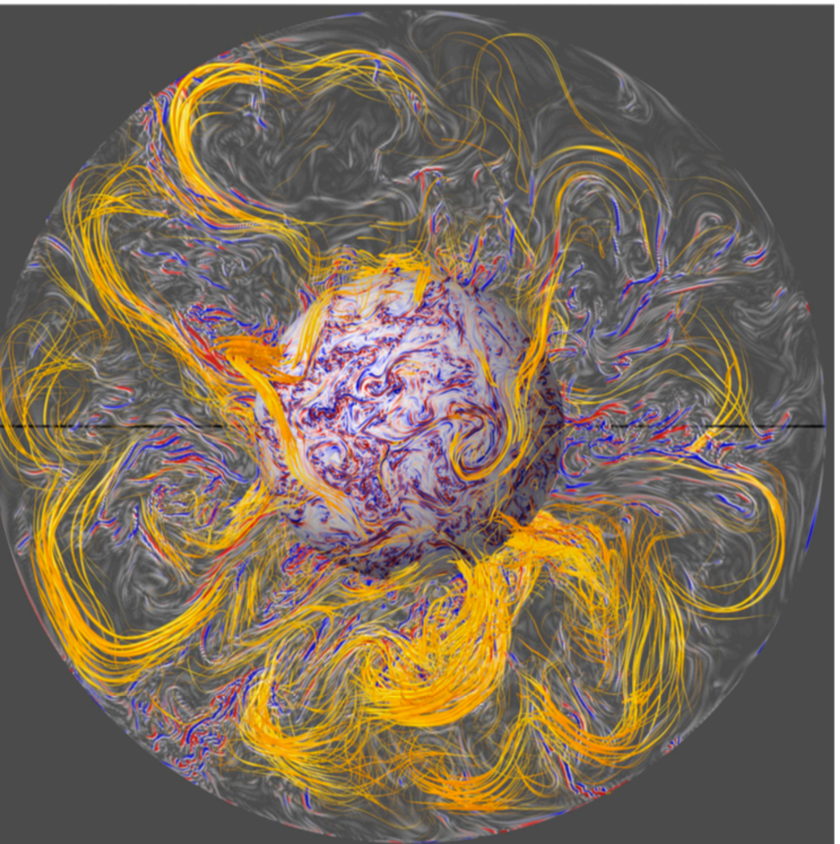
Image: Ed Garnero, Arizona State University

Magnetic Fields, Electric Currents, Motion

Magnetic Fields

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B} \qquad \vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$

Motion



A dynamo is a system that generates a magnetic field, converting kinetic energy to magnetic energy

Aubert (2019)
Geophysical Journal International

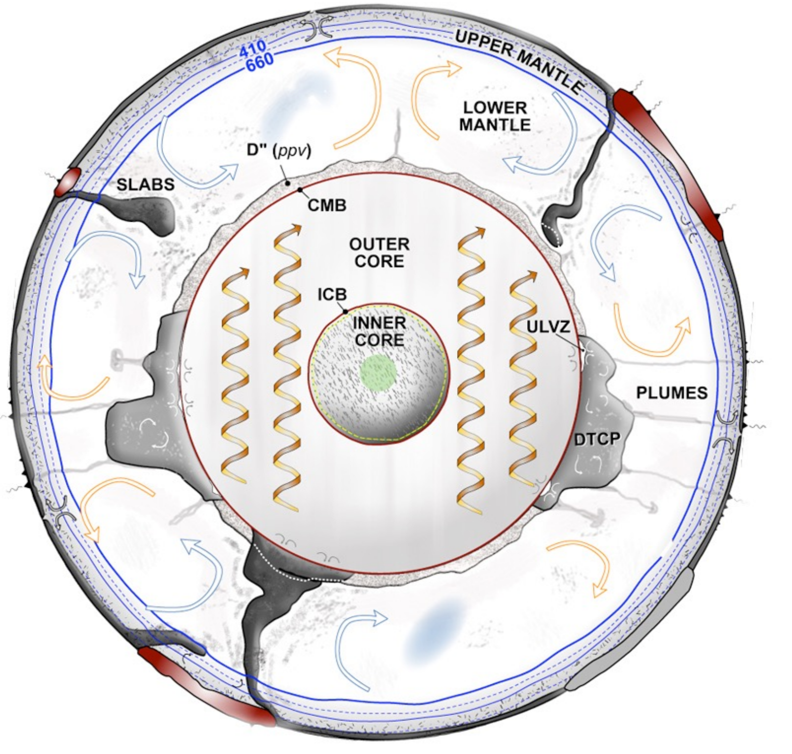


Image: Ed Garnero, Arizona State University

Magnetic Fields, Electric Currents, Motion

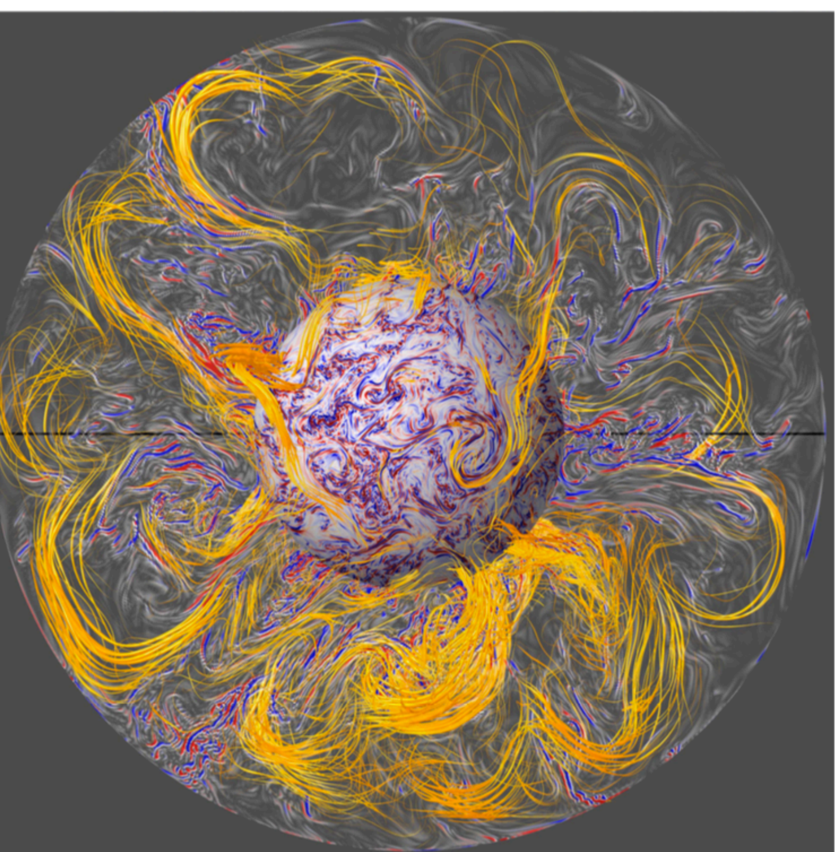
Magnetic Fields

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

Motion

$$\vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$

Electric Currents



Aubert (2019)
Geophysical Journal International

A dynamo is a system that generates a magnetic field, converting kinetic energy to magnetic energy

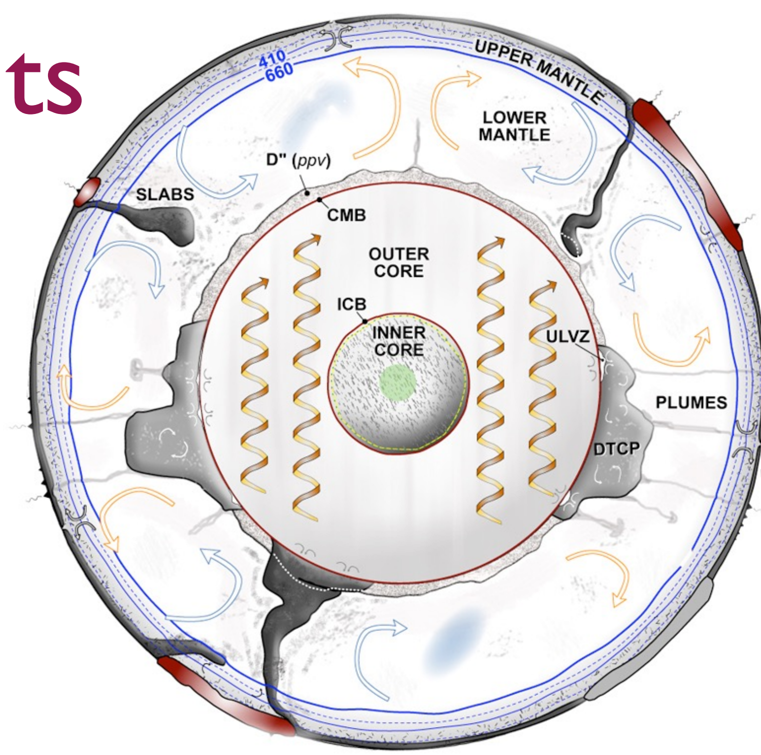


Image: Ed Garnero, Arizona State University

Beautiful Equations?

Acceleration due to forces

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi + \frac{1}{\rho_0}(\vec{j} \times \vec{B}) + \alpha_v \delta T \vec{g} + \nu \nabla^2 \vec{v}$$

Energy conservation

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + \frac{H}{c_p} + \frac{|\vec{j}|^2}{\sigma \rho c_p}$$

Magnetic fields and motion and electric currents

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

$$\vec{j} = \frac{1}{\mu}\left(\nabla \times \vec{B}\right)$$

Beautiful Equations?

Acceleration due to forc

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)(\vec{v}) + 2\vec{\Omega} \times \vec{v} = -\nabla \Pi$$

Image: wikipedia

Beautiful Equations?

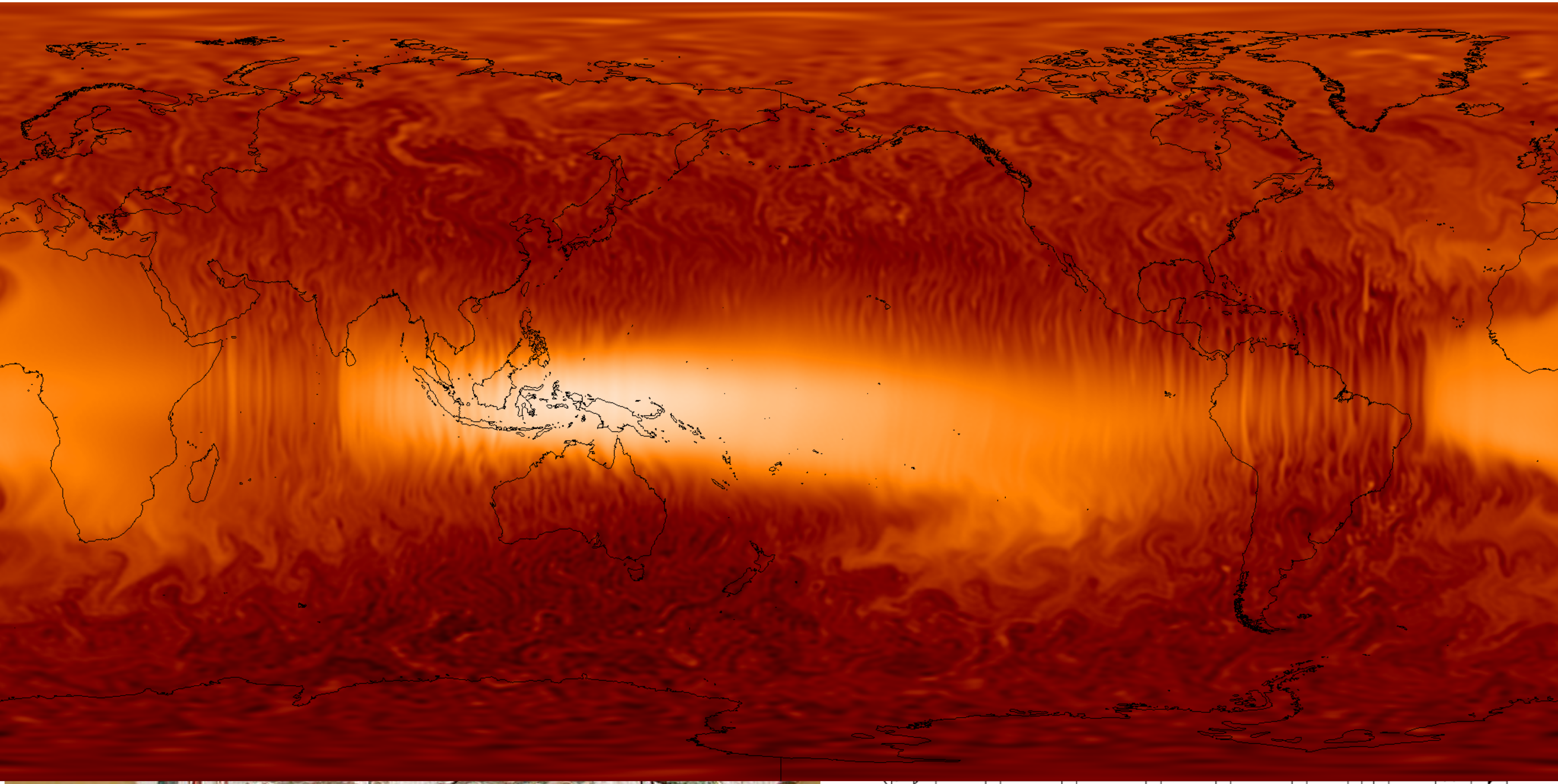


Image: wikipedia